Physics with Extra Dimensions
Lecture I
Flat Extra Dimensions

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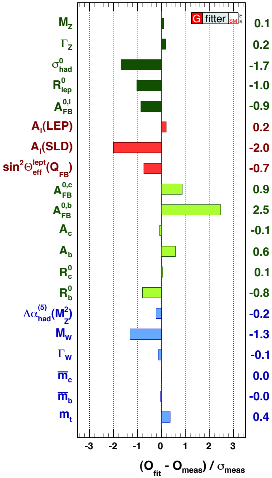
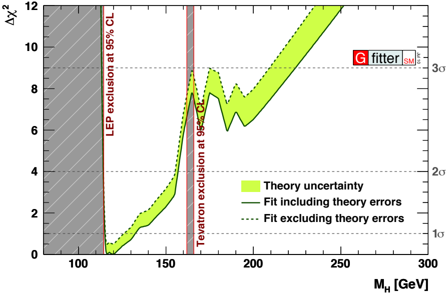
The great success of the SM

The Standard Model success



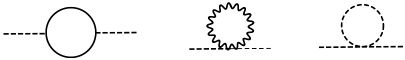
The SM Higgs is light



$m_h < 185$ GeV @ 95% C.L.

The Stability of the Weak Scale

But if Higgs elementary scalar quantum corrections drive m_h up



$$\delta m_h^2 \sim \frac{c^2}{16\pi^2}\,\Lambda^2$$

- We need $\Rightarrow m_h \lesssim 1$ TeV
- But if $\Lambda \to M_P \sim 10^{19}\ GeV$, unnatural

$\Rightarrow$ Gauge Hierarchy Problem

Mechanism to Stabilize the Weak Scale

New physics at $\Lambda \sim 1$ TeV is:

Weakly Coupled

- SM with a light Higgs
- SUSY (MSSM, NMSSM, Folded, ...)
- Little Higgs, Twin Higgs
- LED, UED

Strongly Coupled

- Technicolor, Walking Technicolor
- Topcolor, Top See Saw
- Composite Higgs
- Warped Extra Dimensions

Other reasons to go Beyond the Standard Model

Despite all the sucesses, the Standard Model

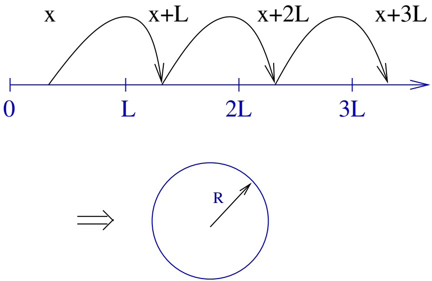
- Does not have a candidate for dark matter
- Does not explain the matter/anti-matter asymmetry
- Does not explain the fermion mass hierarchy
- Does not explain the origin of the Higgs sector nor the stability of the weak scale at $v \simeq 250$ GeV

Theories with Extra Dimensions

- May address some of these problems
- Describe some 4D *Strongly Coupled Theory*

Compact Extra Dimensions

- Extra spatial dimensions with points periodically identified
- 1 Extra Dimension: equivalent to a circle

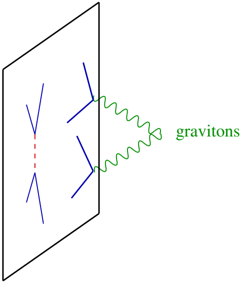


with $R = L/2\pi$. We identified the points

$$x \sim x + L \sim x + 2L \sim x + 3L \sim \cdots$$

Large Extra Dimensions

- Assume space has $3 + n$ dimensions.
- The extra n dimensions are compact and with radius R.
- All particles are confined to a 3-dimensional slice ("brane").
- Gravity propagates in *all* $3 + n$ dimensions.



Large Extra Dimensions

(Arkhani-Hamed, Dimopoulos, Dvali '98)

- Gravity appears weak ($M_P \ll M_W$), because it propagates in large extra dimensions... Its strength is diluted by the volume of the n extra dimensions.
- Fundamental scale is $M_* \sim M_W$, not M_P

$$M_P^2 \sim M_*^{n+2}\, R^n$$

- There is no hierarchy problem:
  The fundamental scale of Gravity

$$M_* \sim 1\ \mathrm{TeV}$$

Large Extra Dimensions

If we require $M_* = 1$ TeV:

$$R \sim 2 \cdot 10^{-17}\, 10^{\frac{32}{n}}\,\mathrm{cm}$$

- $n = 1 \Longrightarrow R = 10^8$ Km. Already excluded!
- $n = 2 \Longrightarrow R \simeq 2$ mm. Barely allowed by current gravity experiments.
- $n > 2 \Longrightarrow R < 10^{-6}$ mm. This is fine.

Large Extra Dimensions - Compactification

- When field propagates in one extra dimension

$$P_M = P_\mu + P_5$$

  with $\mu = 0, 1, 2, 3,\ M = \mu, 5$.

- But XD is compact $\Rightarrow$ P_5 is quantized:
  periodicity $\Rightarrow$ wavelength has to be integer number of $2\pi R$.

$$P_5 = \frac{n}{R}\ , \qquad\qquad (n = 0, 1, 2, 3, \cdots)$$

Large Extra Dimensions - Compactification

- If field has mass M

$$P_M P^M = P_\mu P^\mu - P_5^2 = P_\mu P^\mu - \frac{n^2}{R^2}$$

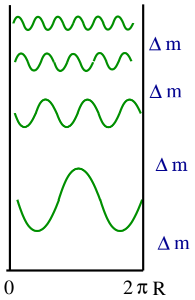
- From the 4D point of view:

$$P_\mu P^\mu = M^2 + \frac{n^2}{R^2}$$

- E.g. for a photon (or graviton) $M = 0$.
  There is a "$n = 0$-mode" with zero mass (our photon/graviton), plus infinite excitations with masses n/R.

Large Extra Dimensions

Compact extra dimensions $\Rightarrow$ graviton excitations (Kaluza-Klein)



Mass gap $\Delta m \sim 1/R$

Large Extra Dimensions

E.g. for

$$n = 2 \longrightarrow \Delta m = 10^{-3}\ \text{eV}.$$
$$n = 3 \longrightarrow \Delta m = 100\ \text{eV}.$$
$$\vdots$$
$$n = 7 \longrightarrow \Delta m = 100\ \text{MeV}.$$

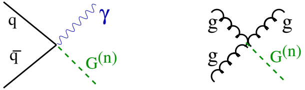
Large Extra Dimensions - Phenomenology

- Individual KK graviton couplings gravitationally suppressed ($\sim 1/M_P$).
- But for $E \gg 1/R \rightarrow$ sum of KK mode results in

$$\sigma \sim \frac{E^n}{M_*^{n+2}}.$$

- Collider Processes:

E.g. Graviton production



Individual graviton decay rates $\sim 1/M_P^2$, $\Rightarrow$ $\not{E}_T$ signals at colliders.

Bounds on M_* from LEP and Tevatron $(1-10)$ TeV.

Universal Extra Dimensions

(Appelquist, Cheng, Dobrescu '01)

- If some SM fields propagate in the bulk $\Rightarrow$ $\boxed{1/R \gtrsim 1 \text{ TeV}}$.
- But if we assume *all* fields can propagate in the extra dimensions. What is the allowed R ?

Universal Extra Dimensions

For example, a scalar field $\Phi(x, y)$ in one extra dimension:

$$S[\Phi(x,y)] = \frac{1}{2}\int d^4x\, dy \left(\partial_M \Phi \partial^M \Phi - M^2\Phi^2\right)$$

- Periodic boundary conditions:

$$\Phi(y) = \Phi(y + 2\pi R)$$

- Expand in Fourier modes:

$$\Phi(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0} \left[\phi_n(x) \cos\left(\frac{ny}{R}\right) + \tilde{\phi}_n(x) \sin\left(\frac{ny}{R}\right)\right]$$

- $\phi_n(x)$ and $\tilde{\phi}_n(x)$ are 4D fields.

Universal Extra Dimensions

- Integrate over the compact dimension:

$$S_{\mathrm{4Deff.}}[\phi, \tilde{\phi}] = \int_0^{2\pi R} dy\, S[\Phi]$$

with

$$\begin{aligned} S_{\mathrm{4Deff.}} &= \sum_{n=0} \frac{1}{2} \int d^x \left[\partial_\mu \phi_n \partial^\mu \phi_n - m_n^2 \phi_n^2\right] \\ &+ \sum_{n=0} \frac{1}{2} \int d^x \left[\partial_\mu \tilde{\phi}_n \partial^\mu \tilde{\phi}_n - m_n^2 \tilde{\phi}_n^2\right] \end{aligned}$$

with

$$m_n^2 = M^2 + \frac{n^2}{R^2}$$

Universal Extra Dimensions

- Momentum conservation in the extra dimensions
  At any vertex, P_M, is conserved.
  Then 4D-momentum conservation $\Rightarrow$ P_5 is conserved.
- E.g.in $(1) + (2) \to (3)$

$$p_5{}^{(1)} + p_5{}^{(2)} = p_5{}^{(3)}$$

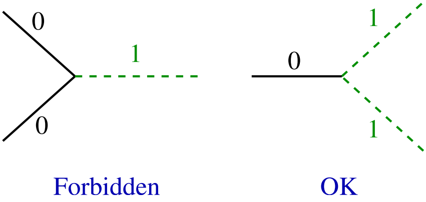
In terms of KK modes, this reads

$$\pm n_1 \pm n_2 = \pm n_3$$

$$\Rightarrow \boxed{\text{KK-number conservation}}$$

Universal Extra Dimensions

For instance,



$\Rightarrow$ KK excitations must be pair produced

This leads to

- Bounds on $1/R$ are lower.
- Distinctive phenomenology

Universal Extra Dimensions - Fermions

The action for a bulk fermion in 5D:

$$S_\Psi = \int d^4x\,dy\,\bar{\Psi}(x,y)\left[i\partial_M\Gamma^M - M\right]\Psi(x,y)$$
$$\int d^4x\,dy\,\bar{\Psi}(x,y)\left[i\partial_\mu\Gamma^\mu - M\right]\Psi(x,y) - \bar{\Psi}(x,y)\gamma_5\partial_5\Psi(x,y)$$

- Clifford algebra in 5D

$$\{\Gamma_M, \Gamma_N\} = 2\eta_{MN}$$

with $\Gamma_\mu = \gamma_\mu$ and $\Gamma_5 = i\Gamma_5$.
$\Rightarrow \Psi(x,y)$ are 4-component Dirac spinors.

Universal Extra Dimensions - Fermions

- After "dimensional reduction" (integrating in y):

$$S_\psi = \sum_{n=0} \int d^4x \left[\bar{\psi}_n \left(i\partial_\mu \gamma^\mu - M + i\frac{n}{R} \right) \psi_n \right]$$

- Zero mode ($n = 0$), is always a vector-like fermion! But in the SM we need chiral fermions!

Universal Extra Dimensions - Fermions

Chirality: Define

$$\Psi = \Psi_L + \Psi_R$$

And ask properties under $y \to -y$ reflections ("parity"):

$$\gamma_5 \Psi(-y) = \pm\Psi(y)$$

Given that

$$\gamma_5 \Psi(-y) = -\Psi_L(-y) + \Psi_R(-y)$$

If we have

$$\begin{aligned} \Psi_R(-y) &= \Psi_R(y) \\ \Psi_L(-y) &= -\Psi_L(y) \end{aligned}$$

then $\Psi_L(x, y)$ is odd, $\Psi_R(x, y)$ is even under parity.

Universal Extra Dimensions - Fermions

- In this case, expanding in KK modes:

$$\Psi(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0} \left[\psi_{nR}(x) \cos\left(\frac{ny}{R}\right) + \tilde{\psi}_{nL}(x) \sin\left(\frac{ny}{R}\right) \right]$$

- So that the zero mode is *RightHanded* !
- Had we chosen $\gamma_5 \Psi(-y) = -\Psi(y)$, i.e.

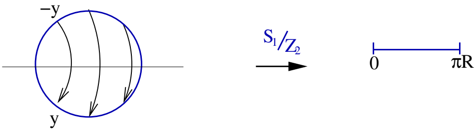
$$\begin{aligned} \Psi_R(-y) &= -\Psi_R(y) \\ \Psi_L(-y) &= \Psi_L(y) \end{aligned}$$

Then the zero mode would be *LeftHanded*.

Universal Extra Dimensions

But how do we define "parity" in a circle ?

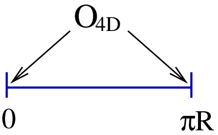
- Orbifold Compactification:
  Identify points opposite in the circle ($y \sim -y$).



- Circle now reduced to segment, with "fixed points" at 0 and πR.
- Fields can be even or odd under $y \to -y$.
- Bulk fermions have chiral zero modes (either LH or RH).

Universal Extra Dimensions

- But Orbifolding breaks KK-number conservation !
  Translation invariance broken in the y direction
  $\Rightarrow p_5$ not conserved !
- The presence of fixed points breaks KK number. By how much ?

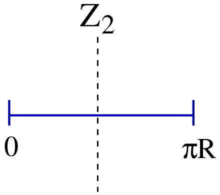


Localized 4D oprators at $y = 0$ and $y = \pi R$ generate KK-number-violating interactions. E.g:

$$S_{\rm loc.} = \int d^4x \int_0^{\pi R} dy\, i\bar{\Psi}(x,y)\gamma_\mu D^\mu \Psi(x,y) \left(\delta(y)\frac{c_1}{\Lambda} + \delta(y-\pi R)\frac{c_2}{\Lambda}\right)$$

Universal Extra Dimensions

- UV physics might not operate differently in $y = 0$ and $y = \pi R$.



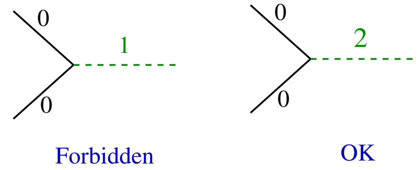
- If $c_1 = c_2 \Rightarrow$ KK-number violating interactions still respect KK-parity.
  E.g. in $(1) + (2) \leftrightarrow (3)$

$$(-1)^{n_1+n_2+n_3} = 1$$

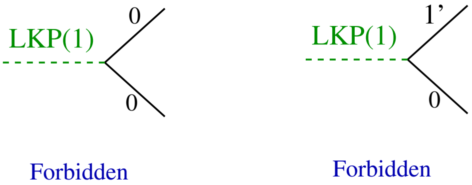
Universal Extra Dimensions

Conservation of KK parity ⇒

- Can produce *level* 2 KK modes in s-channel.



- Lightest KK Particle of level 1 (LKP) is stable



⇒ LKP is Dark Matter candidate

UED Phenomenology

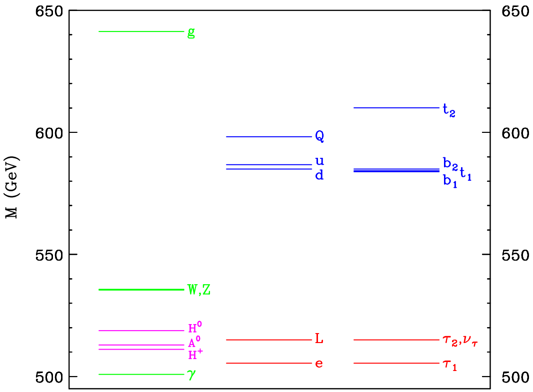
- Electroweak precision constraints:

$$1/R \gtrsim 300 \text{ GeV for 5D}$$
$$1/R \gtrsim (400 - 600) \text{ GeV for 6D}$$

- Current direct searches give similar bounds.

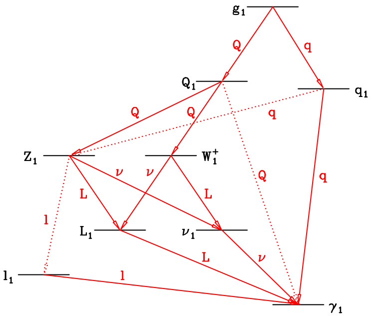
UED Phenomenology

- Spectrum at each KK level is degenarate at tree level. Localized operators split the masses (one-loop generated).
- First KK mode in 5D model, with c_i's computed at one-loop.



UED Phenomenology

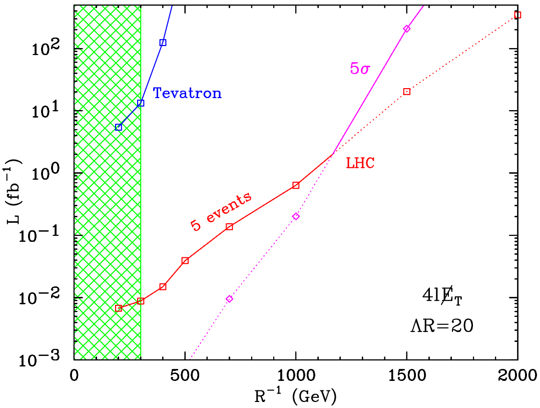
- Light KK modes $\Rightarrow$ large cross sections.
- But, almost degenerate KK levels $\Rightarrow$ little energy release.



Best mode $q\bar{q} \to Q_1 Q_1 \to Z_1 Z_1 + \not{E}_T \to 4\ell + \not{E}_T$ (Cheng, Matchev, Schmaltz '02).

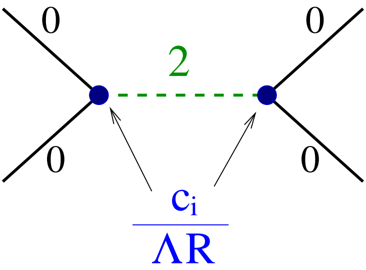
UED Phenomenology

Reach using this golden mode $q\bar{q} \to\to 4\ell + \not{E}_T$



UED Phenomenology

- Production and Decay of Second KK Level:
- They couple to 2 zero modes through brane couplings (loop generated). (Datta, Kong, Matchev '05



with $\Lambda R \gg 1$ and $c_i \sim O(1)$.

- But has to compete with $2 \to 1 + 1$

UED 6D Phenomenology

- Signals very different in 6D (Burdman, Dobrescu, Pontón '06)
- More scalar degrees of freedom: E.g. A_M, with $M = 0, 1, 2, 3, 5, 6$
- In 5D A_5 not physical. Eaten by KK modes to get their masses (NGB of breaking of translation invariance).
- In 6D one linear combination of A_5 and A_6 is eaten, but one remains in the spectrum
- LKP is a scalar: $B_{5,6}$, extra component of 6D hypercharge gauge boson
- $\Rightarrow$ Scalar dark matter candidate.

UED Model Building

Some remarks:

- Standard UED models do not solve the hierarchy problem
- Theories with compact extra dimensions can be viewed as (dual to) 4D strongly coupled theories:
  - KK modes $\leftrightarrow$ hadron excitation spectrum
  - Inverse of compactification radius R^{-1} $\leftrightarrow$ Excitation gap in strong interaction: Λ (Eg. $\Lambda_{QCD} \simeq 1$ GeV)