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Entropy-based relativistic smoothed particle hydrodynamics

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Abstract
The smoothed particle method for hydrodynamics (SPH) is extended to describe the process of ultra-relativistic heavy ion collisions. Equations of motion have been derived from the variational principle. In order to treat the baryon free matter, entropy is taken as the basis of the SPH representation. This approach is very suitable for the event by event analysis of the hydrodynamical evolution of the collision.

1. Introduction

One basic point in the hydrodynamic approach to RHIC physics is that its principal ingredients, i.e. the equation of state of the matter and the initial condition for the dynamics are not quite well known. In contrast to this approach, we apply the hydrodynamic models to infer this information, that is, the properties of the matter in such a highly condensed and excited state. Therefore, we actually do not need a very precise solution of the hydrodynamics but a general flow pattern which characterizes the final configuration of the system as the response of a given set of an equation of state and an initial condition. We are not interested in very precise local features in these models. Extremely local properties thus should be averaged out without spoiling the general flow pattern. This is particularly the case when one considers the questionable validity of the local thermal equilibrium in the problem of interest here. In summary, for the study of hydrodynamic models of relativistic nuclear collisions, we prefer a rather simple scheme for solving the hydrodynamic equation, not unnecessarily precise but robust enough to deal with any kind of geometry. From this point of view, the advantage of the variational approach to relativistic hydrodynamic models is stressed in [1].

Among the many numerical approaches, the smoothed particle hydrodynamics (SPH) approach [3–5] fits perfectly in a variational formalism and presents many desirable profiles of the variational approach, such as simplicity and robustness with respect to the changes of geometry, as well as the possibility of smoothing out undesirable local degrees of freedom. Furthermore, the SPH parametrizes the matter flow in terms of discrete Lagrangian coordinates (called ‘particles’), so that it is best for studies of relativistic nuclear collisions, where extremely compressed and high-temperature hadronic matter expands into a very large space region.

Several studies of the SPH algorithm for the ultrarelativistic regime of hydrodynamics have been made [2]; however, some specific aspects of the relativistic heavy-ion collision processes deserve some attention when we apply the SPH. One of them is that, in the ultrarelativistic regime of central collisions, a large amount of the incident energy is converted into produced particles. In particular, in the mid-rapidity region of central collisions, most of these energies are in the form of produced pions and only a very small portion is carried by baryons. This is a very unfavourable situation for the conventional formulation of the SPH algorithm, where particles are associated with the matter defined by the conserved quantity, such as the baryon number. The direct application of the SPH based on the conserved baryon number may fail in the region for a null baryon number pion gas. We derive the relativistic SPH (RSPH) equations using the variational principle [1], taking the matter flow as the variational variable. We argue that the quantity we attribute to the SPH particles convenient for relativistic heavy-ion collisions will be the entropy of the fluid. In this way, we can follow directly the entropy content and its change due to the dissipation mechanism, for example, the shock wave. This is particularly interesting for the system where the first-order phase transition is present [6–8].

2. Entropy-based RSPH and the variational procedure

The basic point of the SPH representation is that we parametrize the density n^* of a conserved quantity, such as the baryon number by the following ansatz:

$$n^*(\vec{r}, t) = \sum_i \nu_i W\left(\vec{r} - \vec{r}_i(t); h\right) \tag{1}$$

where $W\left(\vec{r} - \vec{r}\,'; h\right)$ is a positive-definite kernel function peaked at $\vec{r} = \vec{r}\,'$ with the normalization $\int \mathrm{d}^3\vec{r}\; W\left(\vec{r} - \vec{r}\,'; h\right) = 1$. The parameter h represents the width of the kernel. The role of W with a finite value of h is to introduce a sort of short-wavelength cut-off filter in the Fourier representation of the density n^*. The velocity of these particles are identified as the velocity of the fluid at their position $\vec{r}_i(t)$, $\vec{v}_i = \frac{\mathrm{d}\vec{r}_i}{\mathrm{d}t}$ so that the Lorentz factor of the ith particle is given by $\gamma_i = 1/\sqrt{1 - \vec{v}_i^2}$. If N is a conserved quantity, then $\{\nu_i\}$ should be constant in time. We consider the set of time-dependent variables $\{\vec{r}_i, i = 1, \ldots, n\}$ as the variational dynamical variables and their equations of motion are determined from the action for the hydrodynamic system. Here, $\{\nu_i\}$ are not dynamical variables and are determined by the initial condition together with the constraints for the variational procedure. The relativistic hydrodynamic equation can be obtained by the variational principle [1] and the corresponding Lagrangian for the system of SPH particles can be taken to be

$$L_{SPH}\left(\left\{\vec{r}_i, \frac{\mathrm{d}\vec{r}_i}{\mathrm{d}t}\right\}\right) = -\sum_i \left(\frac{E}{\gamma}\right)_i \tag{2}$$

where $\{\vec{r}_i(t)\}$ are the dynamical variables and E_i is the 'rest energy' of the particle i which is related to the energy density ε_i as $E_i = \nu_i \varepsilon_i / n_i$.

As we have mentioned in the introduction, in the application of hydrodynamics to the ultrarelativisitic nuclear collisions the baryon number is not a suitable quantity for representing the energy flow since most of the energy content is in the form of non-baryonic matter. This is particularly so in the central region of rapidities. We may consider the energy content itself as the SPH base. However, as mentioned before, this choice introduces an additional constraint between the coordinates $\{\vec{r}_i\}$ and the extensive parameter $\{\nu_i\}$ of the SPH particles due to the conservation of energy and is not desirable from a practical point of view. Therefore, we propose the entropy as being a suitable extensive quantity for the SPH representation.

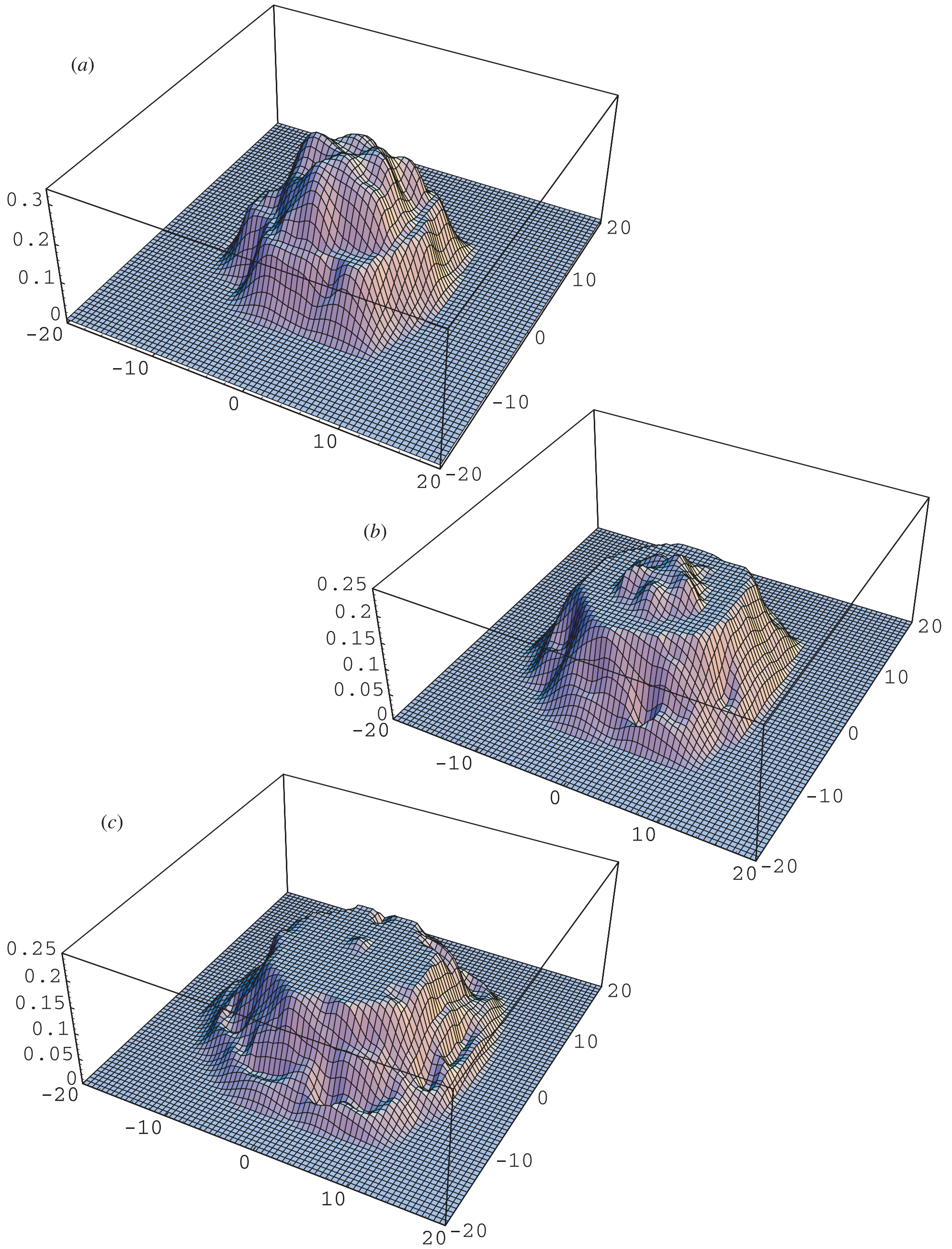

Figure 1. NeXus2β + SPheRIO (Au + Au 100 GeV + 100 GeV) temperature (GeV) in the x–y (fm) plane ($\eta = 0$). (a) $\tau = 1$ fm (initial); (b) $\tau = 3$ fm; (c) $\tau = 5$ fm.

(This figure is in colour only in the electronic version, see `www.iop.org`)

Let $s^*(t, \vec{r})$ be the entropy density in the space-fixed (calculational) frame. We then express it as $s_i^* = \sum_j \nu_j W\left(\vec{r}_i - \vec{r}_j\right)$. In this case, ν_i is the amount of entropy carried by the

ith particle. The Euler–Lagrange equation of motion for $\{\vec{r}_i\}$ leads to

$$\frac{d\vec{\pi}_i}{dt} = -\sum_j \left[\frac{\nu_i}{s_i^{*2}}(P+Q)_i + \frac{\nu_j}{s_j^{*2}}(P+Q)_j \right] \nabla_i W_{ij} \quad (3)$$

where P is the pressure and $Q = Q(N, S, \theta)$ is the dissipation pressure related to the bulk viscosity used to express the entropy production process. $\vec{\pi} = \nu\gamma\vec{v}\,(P + Q + \varepsilon)/s$ is the momentum density associated with the ith particle. We have

$$\frac{1}{\nu_i}\frac{d\nu_i}{dt} = -\frac{Q_i\gamma_i}{T s_i^*}\theta_i \quad (4)$$

where

$$\theta_i \equiv \frac{1}{V_i}\frac{dV_i}{dt} = \partial_\mu u^\mu. \quad (5)$$

Equations (3) and (4) together with $d\vec{r}_i\, dt = \vec{v}_i$ constitute a system of the first-order differential equations for $\vec{r}_i$, $\vec{\pi}_i$ and ν_i.

3. Discussion and perspectives

In the usual space grid hydrodynamic approach, the symmetry of the problem is often a crucial factor in performing a calculation of reasonable size. The SPH method cures this aspect and furnishes a rather robust algorithm which is particularly appropriate for the description of the expansion phenomena. Here we formulated the entropy-based relativistic SPH description of the relativistic hydrodynamics. In figure 1 we show an example of the present approach for the initial condition obtained by the NEXUS code by Klaus Werner. Here, we assume a very simple bag-model-type baryon-free equation of state and we observe the development of a plateau in the temperature. As we can see the present approach is very promising for the study of ultrarelativisitic nucleus–nucleus collision processes. Because of the variational aspects based on the Lagrangian comoving coordinates, we expect a maximal efficiency for a given number of degrees of freedom. Therefore, a very reasonable precision for the solution can be obtained with a relatively small number of SPH particles. This is the basic advantage of the present method.

Acknowledgments

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