SPheRIO manual

(Dated: September 19, 2010)

SUMMARY

This document is the programing manual for the FORTRAN77 version of SPheRIO, which implements the entropy representation of the Smoothed Particle Hydrodynamics (SPH) method for relativistic high-energy collisions. The acronym SPheRIO stands for Smoothed Particle hydrodynamical evolution of Relativistic heavy-IOn collisions, therefore being also a reminder that the code has been developed within the São Paulo - Rio de Janeiro Collaboration.

Section I gives a bird-eye's view of the program structure of SPheRIO. A complete explanation of the option file OPTNS can be found in section II. Section III is devoted to explain the conventions and interfaces used in the Equation of State (EOS) tables. Section IV focuses on the details of functions and algorithms of each subroutine. In Section V, some simple but practical examples are given to show how one may connect his own Initial Conditions (IC) or EOS with SPheRIO by complying with the code's interface standard.

Contents

I. MAIN PROGRAM STRUCTURE

SPheRIO is an implementation of hydrodynamic model of nucleus-nucleus collisons based on the SPH method. The main reference to SPheRIO is

Y. Hama, T. Kodama and O. Socolowski Jr., Braz. J. Phys. 35 (2005) 24,

and to the Smoothed Particle Hydrodynamics (SPH) method adopted in SPheRIO is

C.E. Aguiar, T. Kodama, T. Osada, Y. Hama, J. Phys. G27 (2001) 75.

The present release of SPheRIO (v4.01) was built in such a way that it can be easily used and manipulated by someone who is unfamiliar with the source code. On the other hand, an experienced programmer can further develop the code for inserting SPheRIO into another program without much effort. In its current form, SPheRIO consists of several blocks as shown in Fig.1.

Each block possesses its specific functionalities, meanwhile it can be considered as almost independent from the rest of the code. In fact they can be compiled independently and only communicate with other parts of the code through well defined interfaces. Therefore, debugging one of the blocks in principle should not affect the remaining parts. Several blocks were implemented so that they could be replaced by the user's own codes without much effort, by simply implementing some interface protocols explained in this document. Concerning the output format of the code, we decided that it should be determined in such a way as to be sufficient and convenient to connect to other open source codes which deal with particle decay and rescattering. Within this spirit, we provide several different output formats.

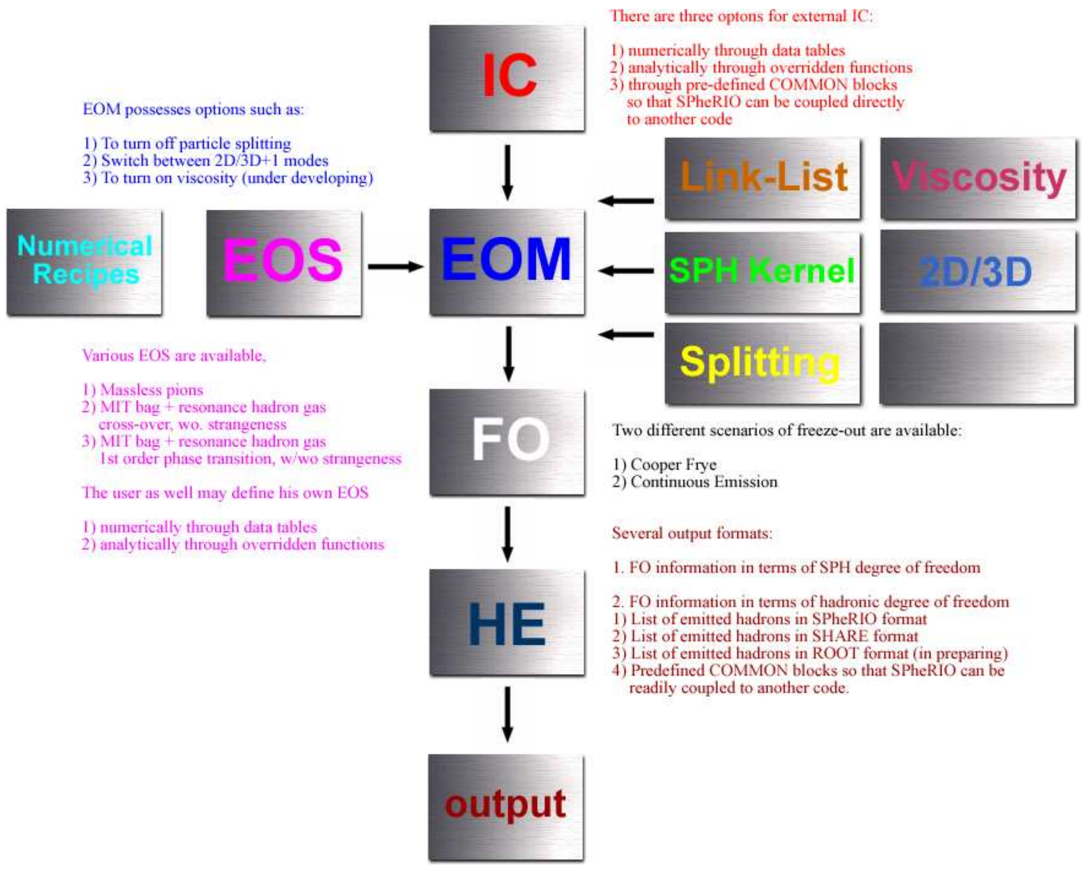
This section is mainly devoted to explain the structure of SPheRIO and the basic ideas of its communication interface.

The code SPheRIO is designed to simulate the hydrodynamic evolution in nucleus-nucleus collisions. Its Equation of Motion (EOM) is based on the SPH method. For practical reasons, the master routine of SPheRIO, namely, **spherio()** [1] is embodied in a small shell script, namely, **aamain()**. It is the place where most of the initializations are done and the parameters are read and interpreted by the subroutine **aread()**[2]. From the shell, SPheRIO is called not once but *nevent*[3] times, where $nevent = nfreeze \times nfull$, $nfull$ is the number of full events, and $nfreeze$ is the number of Monte Carlo (MC) freeze-out procedures which emulate the hadron emission. When the code is run in an Event-by-Event (EbyE) basis, $nfull$ should be defined as the number of total events, i.e., $nevent = nfull$. Effectively, one may consider two loops, the outer loop goes from 1 to $nfull$ and, for each outer loop, the inner loop goes from 1 to $nfreeze$, so that each full collision event possesses $nfreeze$ MC processes. To be more specific, each time one begins an outer loop, **spherio()** is evoked. It then calculates the full hydrodynamic evolution from scratch, storing all the information on the freeze-out surface, after which the hadron emission is then carried out. By means of

[1] the subroutines and input files used in the SPheRIO code are highlighted in boldface characters in this manual

[2] The subroutine **aread()** adopts NEXUS/EPOS input technique to a great extent; we are very thankful for the help from Prof. Klaus Werner.

[3] Variable names and keywords are denoted by italic letters whenever they appears in the text



There are three optons for external IC:

1) numerically through data tables
2) analytically through overridden functions
3) through pre-defined COMMON blocks
so that SPheRIO can be coupled directly
to another code

EOM possesses options such as:

1) To turn off particle splitting
2) Switch between 2D/3D+1 modes
3) To turn on viscosity (under developing)

Various EOS are available.

1) Massless pions
2) MIT bag + resonance hadron gas
cross-over, wo. strangeness
3) MIT bag + resonance hadron gas
1st order phase transition, w/wo strangeness

The user as well may define his own EOS

1) numerically through data tables
2) analytically through overridden functions

Two different scenarios of freeze-out are available:

1) Cooper Frye
2) Continuous Emission

Several output formats:

1. FO information in terms of SPH degree of freedom

2. FO information in terms of hadronic degree of freedom
1) List of emitted hadrons in SPheRIO format
2) List of emitted hadrons in SHARE format
3) List of emitted hadrons in ROOT format (in preparing)
4) Predefined COMMON blocks so that SPheRIO can be
readily coupled to another code.

FIG. 1: Block diagram of SPheRIO programming structure.

xellwrite() the results are directed to the output file or to other subroutine to be added. Starting from the second inner loop, SPheRIO only computes hadron emission based on the results of the hydrodynamic evolution that it had already obtained and stored in the freeze-out surface. The successive inner loops repeat the process of the second inner loop until a new outer loop starts, where some (presumably) different IC are put into action. This means that several distinct hadronic emission processes use the same information stored on the freeze-out surface at the end of the hydrodynamic evolution, for saving computer time. The double loop structure ends when it finally enumerates all outer loops.

To discuss the general structure of the code, a good place to start with is the master routine **spherio()** itself, since many important subroutines of SPheRIO are invoked there. These subroutines appear in the code in the same order as the run-time calculations develop. It contains the following major blocks, as shown in Fig.1:

- Initial conditions (IC)
- Equation of Motion (EOM)

- Freeze-out (FO)
- Hadron Emission (HE)
- FO info output
- Radius obtained from identical particle correlations or Hanbury-Brown & Twiss effect (HBT)

There are some other blocks which have distinct features or independent functionalities, however they are written in such a way that they are either partially integrated within or frequently invoked from different parts of the code. Therefore it is preferable to separate them from the rest of the code and treat each of them individually. Those blocks are:

- Link-list method
- Particle splitting (it is literally called particle decay in the code)
- Kernel functions
- Equation of state (EOS)
- Pure numerical recipes

Last but not least, to debug SPheRIO, one also has to know the following aspects

- Input interface between SPheRIO and end-users
- SPheRIO programming style

Master routine

spherio()

The master routine **spherio()** obtains parameters handed down from the shell script through the **optns** file as well as through **spherio.set** and **spherio.inc**. The values of some parameters are defined for a second time according to the setting of $ibugflg$. It is a convention in the code that those values determined via $ibugflg$ settings have a high priority than those defined by **optns** file or **spherio.set**. The reason is that it provides a way to set up a group of parameters in a single step, as specific scenarios for debugging or for other practical purposes. Therefore one doesn't have to go through every single parameter contained in the option file which might be error prone. As an example, we have

◇ $ibugflg = 08$ defines an environment to run the code with EOS of massless pions in the 2D+1 case, namely, the EOM will be solved only on the x-y plane, and using the Bjorken scaling solution for the η direction. At the end, the code will print out the profile of the hydrodynamic evolution of the system at pre-defined time instants $\tau = 1, 3, 5, 10$. Since this setting is frequently used, it is more convenient to simply set $ibugflg = 08$ to activate above set of pre-defined parameters instead of set by hand their values one by one in the **optns** file, which would require to use “set *ieos* 04”, “set *itrobmode* 01”, “set *idimension* 2”, etc.

The master routine **spherio()** creates and controls the main surveillance file **ztr.data**, which prints the de facto value of all the major parameters at run-time and monitors the execution of program with necessary debugging outputs and warning messages. Depending on the parameter *ieos*, **spherio()** loads correspondingly the EOS tables, and some preliminary tests are made. An energy density rescaling file might be applied at this point accroding to certain parameters. Typically, **config()** or **config_pion()** is called to initialize the lattices, SPH particles are consequently created and assigned to the grids with their respective velocity, energy and entropy values. Then the time loop begins to calculate the hydrodynamic evolution. Here we come across two important subroutines: **solvediff()** and **lssf()**. They are the meat and bread of SPheRIO. The subroutine **solvediff()** deals with the EOM, using 2^{nd} order Adams-Moulton method; **lssf()** handles the freeze-out. These two subroutines have to be invoked at the same time, and each one depends on the result of the other. This is because the freeze-out needs the information on the EOM, and this might be affected by freeze-out when frozen or low temperature SPH particles are taken away from the fluid. At the end of the hydrodynamical evolution, **setevg()** and **evgen()** are invoked to treat the hadron emission and to pass the information back to the shell script. The HBT radius is computed by **evghbt()**, as needed.

Initialization

The initialiation block, as the name self-explains, includes various subroutines which carry out the initialization work of the code, for instance, to interpret the parameters of the configuration file, to load appropriate EOS tables, to divide the space-time into lattice sites, to assign initial value to physical quantities, to define the running debug level, etc.

The block contains the following subroutines: **config(), config_pion(), latt(), loadeos(), machconeini(), modeprint(), newvel(), rfactor(), rfactorprint(), rfactorset(), rfactor_*(), seteosmode(), set_flagce(), set_flagxn(), setin(), setin_kodama(), setin_pion(), setkappa(), setlog(), settautrans(), settempsph(), settempsphdenovo(), settrinimode()**

The subroutine **loadeos()** loads the EOM tables. The role of the subroutine **config()** or **config_pion()** is to divide the space-time into discrete lattices, each of which further calls **latt()**. On the other hand, **latt()** has in itself a loop enumerating all the lattice sites. This is when **setin()**, or **setin_kodama()**, or even **setin_pion()** is executed to assign dynamical quantities (four-velocities) and thermodynamic quantities (entropy density, energy density, volum, etc.) to the corresponding SPH particles sitting at each lattice site. Other subroutines handle the initilization of additional parameters, such as freeze-out temperatures, rescaling factor, initial transverse velocity, as well as to various flags.

The code **SPheRIO** provides three different methods for defining the IC.

◇ The first one is to use the current setting of lattice sites. The user may define

- $nxico, nyico, nzico \leftrightarrow$ number of sites,
- $xminico, xmaxico, yminico, ymaxico, zminico, zmaxico \leftrightarrow$ initial size of the system, and
- $tempoico \leftrightarrow$ initial time τ_0 of hydrodynamic evolution.

Next, one should provide a file which contains the information on energy density ($IcoE$), flavor density ($IcoF$) and velocity distribution ($IcoV$) on the lattice sites. The name of the file can be passed to the code by the keywords: "fname ico $desired\_filename$". The structure of the file can be easily inferred from the transcript subroutine below, which would read it.

```
open(97,file=fnio(1:nfnio),status='old')
read(97,*) iversn !Version of the file, irrelevant
read(97,*) laprojx,maprojx,latargx,matargx
!Atomic number and Mass of the incident nucleus and the target
read(97,*) engyx !Incident energy
read(97,*) bminimx,bmaximx,ikolmnx,ikolmxx
!Maximal and minimal value of impact parameter
read(97,*) tauicox !Initial time for hydrodynamic Evolution
read(97,*) iabs_ninicon !Irrelevent, for compatibility with EPOS only
read(97,*) nxicox,nyicox,nzicox !Number of sites in x y z directions
read(97,*) xminicox,xmaxicox,yminicox,ymaxicox,zminicox,zmaxicox
!Initial size of the system
read(97,*) (((IcoE(ix,iy,iz),ix=1,nxico),iy=1,nyico),iz=1,nzico),
((((IcoV(i,ix,iy,iz),i=1,3),ix=1,nxico),iy=1,nyico),iz=1,nzico),
((((IcoF(i,ix,iy,iz),i=1,3),ix=1,nxico),iy=1,nyico),iz=1,nzico)
close(97,file=fnio(1:nfnio),status='old')
```

To explicitly verify if SPheRIO knows that the IC file is provided, the keyword "set *icomode* 00" should be used. If nothing is ever assigned to *icomode*, the default value is 0.

◇ The second method is to pass the variables through COMMON blocks defined in *epos.incico*[4]. To make use of this "dangerous" option, the users should compile their own code which implements the COMMON block and link it to SPheRIO (through Makefile). This alternative makes it easier to connect SPheRIO directly to an event generator. In this way, one does not need to write the IC into a file and later make SPheRIO read it. It is more efficient, especially when one uses event by event IC. To inform SPheRIO that the IC is passed using this method, use keyword "set *icomode* 01". **Warning**: if *icomode* is set to 01, but nothing is implemented for the COMMON block, there is no way that SPheRIO could figure this out by itself, and **in this case no error message will be triggered directly**.

◇ The third method is to override the subroutine **config_override()** of SPheRIO. It gives the user the biggest freedom to define the IC from scratch. For instance, SPH particles don't have to be initially distributed uniformly in space. As a price to pay, the user must explicitly define how to divide the lattice sites and also initialize and distribute SPH particles by hand. Since the purpose of this subroutine is to be overridden by the user, the only communication between the subroutine and the rest of the code is the interface of the subroutine, and it is not recommend to utilize COMMON blocks to pass data. The present form of the subroutine gives an example of what quantities are expected to be defined in it. To inform SPheRIO that IC is provided by overriding **config_override()**, the keyword "set *icomode* 02" should be used.

[4] The file is called epos.* since it adopts the standard used in EPOS

Hydrodynamic evolution

This block has two purposes. Firstly, to solve the hydrodynamic equation, i.e., Eq.(100) of "Topics on hydrodynamic model of nucleus-nucleus collisions", Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264]. Secondly, to maintain the SPH particle data in itself, and other blocks such as freeze-out, particle splitting, link-list. etc. The code employs 2^{nd} order Adams-Moulton method (though it is named after Runge-Kutta, by a mistake) to solve the second order ordinary differential equation. A nonlinear equation of $\beta = ln(\sqrt{1+u_T^2}+u_T)$ is solved to obtain the four velocity (this is not mandatory, since one may derive the EOM directly in terms of four velocity). The variable chosen to be solve is $\beta = \ln(\sqrt{1+u_T^2}+u_T)$ Since the entropy ν_i appears on both sides of the EOM, it is canceled out. For convenience, the canonical momentum q_i is defined as π_i/ν_i.

The block contains the following subroutines: **dqfn(), dqfn_xy(), findbt(), findbt_xy(), firststep(), fldobsrv(), fldobsrv_fin(), outp_raw(), rung(), solvediff(), tranisotropy(), trobsrv(), trobsrv_old(), trobsrv_pion()**

The subroutine **dqfn()** is used to calculate the r.h.s. of the above mentioned Eq.(100). The subroutine **rung()** implements the 2^{nd} order Adams-Moulton method, which makes use of numerical recipe **rtflsp1()**. At first step, taken for the first time, **firststep()** is invoked to prepare the loop. And **solvediff()** controls overhead. To sum up, **solvediff()** invokes **firststep()**, **rung()**, and **dqfin()** to solve the EOM, while maintenance of data is taken care by itself. Two important subroutines **dectrl()** and **dectrl_off()**, are embodied in **solvediff()**, which we will come back into the discussion later, when discussing particles splitting block.

Freeze-out

The freeze-out block implements different freeze-out scenarios, it calculates the freeze-out surface $d\sigma_\mu$ among other things.

The block contains the following subroutines: **checkce(), checknu(), checkxn(), flintp(), lssf(), normal(), normal_ce(), probce(), probce_gen(), recordxn(), recordxn_inspector()**

lssf() is the meat and potatoes, i.e., the fundamental ingredients, of the freeze-out block. The subroutine **lssf()** describes the Cooper Frye freeze-out scenario within itself. It calls **checkce()** to handle the continuous emission, and **checkxn()** to implement the chemical freeze-out.

Hadron emission

The hadron emission block involves two parts. The first part uses statistical distribution functions to estimate the number of each hadrons species. The subroutines responsible for the first part are **peso()**, **weigpr()** and **setevg()**.

weigpr() calculates the emission number of a specific hadron from a certain SPH particle. **peso()** does pretty much the same thing with some approximation. The approximation in **peso()** reads that the normal vector of freeze-out surface is timelike, therefore its module is the volume of corresponding SPH particle. **setevg()** stores those obtained value.

The second part employs random number generator to produce the hadronic characteristics, such as momenta, and therefore it is ready for the data to be directed to the output subroutine or be passed to another code (e.g. it can be either NEXUS or THERMINATOR) to do hadronic decay (Consult the next section for more details). These subroutines are **prodis()**, **pdmax()** and **evgen()**.

prodis() calculates the probability of the emission of a certain hadron with a specific momentum, where **pdmax()** is employed to find its maximum. Both two subroutines are invoked by **evgen()** together with the random number generator, it enumerates all SPH particles as well as different species of hadrons, in other words, it generates the event as the name self-explains.

One thing worth mentioning is that when HBT radius is calculated, the second part is swtiched off to save CPU time, since this part of code will repeatedly be executed many times when $nfreeze$ is big. When estimating the HBT radius a particular method (see below) is used to deal with hadronic emission.

Freeze-out infomation output

SPheRIO provides several different methods to output the information on the freeze-out surface. The first catalogue supplies the freeze-out surface itself. For the time being, SPheRIO only provides the freeze-out information in terms of the SPH degrees of freedom. To activate this method use "write $fzoutTable$" in the OPTNS file, and utilize "fname $fzout$ $the\_desired\_filename$" to define the filename. On the contrary, use "read $fzoutTable$" to read freeze-out surface information from a file, in this case, the hydrodynamic evolution will be skipped by SPheRIO.

The second catalogue is of greater utility, since it prints out a list of hadrons emitted at the freeze-out surface. Since there are several conventions for programatic particle identification, SPheRIO makes some effort to implement the most popular conventions. Therefore, first use "switch $xell$ on" to switch the output on. Set "set $ifzlistfmt$ 01" for using SPheRIO particle table convention. In this convention, hadrons are identified using SPheRIO ID. Use "set $ifzlistfmt$ 02" for NEXUS/EPOS particle table convention. In this convention, hadrons are identified using NEXUS/EPOS ID, and the output file meets the standard of NEXUS/EPOS. Use "set $ifzlistfmt$ 03" for THERMINATOR/SHARE particle table conventions. In this convention, hadrons are identified using THERMINATOR/SHARE ID, and the output file shall be processed by a slightly modified version of THERMINATOR/SHARE. Use "set $ifzlistfmt$ 04" for ROOT particle table convention (under development). Emitted hadron list will also be available through the COMMON block in the near future.

HBT radius

The HBT block calculate HBT radius using the information on the freeze-out surface.

The block involves the following subroutines: **corr(), evghbt(), fndk(), raplist()**

Specifically, Eq.(120) and Eq.(122) of "Topics on hydrodynamic model of nucleus-nucleus collisions" Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264] are used to calculate the two particle correlation function $C_2(q, P)$. A link-list method (see below) in rapidity space is implemented in the subroutine **raplist()**.

Link-list method

The goal of this block is to provide an efficient way to calculate any dynamical or thermodynamical quantity at any given point.

It contains the f **bsqd(), bsqd_xy(), fins()fins_xy(), finsb_gen(), finsb_gen_xy(), mklt(), prcn()**

The method can be found in the book "Numerical Recipes: The Art of Scientific computing 3rd Edition" Cambridge University Press, ISBN-13 978-0-511-33555-6/ISBN-13 978-0-521-88068-8. The subroutine **mklt()** builds the link-list, **prcn()** picks up nearby SPH particles using the link-list, while among others, **finsb_gen()** calculate the quantities at a given point.

Particle splitting

Particle splitting is introduced because the SPH particles might not be evenly distributed in space due to the hydrodynamical evolution. We should note that the kernel function we employ expands $4hxy$ where hxy and $heta$ measure the size of SPH particle on the transverse and longitudinal direction. If in a cube of $(4hxy) \times (4hxy) \times (4het)$, the number of SPH particles is smaller than 64 (this estimate is not exact, though), the smoothness of SPH formulation is questionable, especially when some SPH particles carry large entropy (However, it is arguable whether it is the right way to heal the problem; an alternative option is to make every SPH carry the same amount of conserved quantity, e.g., entropy, and therefore no decay is necessary [5]. Initially, the SPH particles are not uniformly distributed, so that the number density of SPH particles is proportional to entropy density. In this case, sometimes a very large number of SPH particles is needed to achieve the desired resolution).

decay(), dectrl(), dectrl_off()

The subroutine **dectrl()** determines when the SPH particle splitting takes place, **decay()** implements the splitting. On the other hand, the subroutine **dectrl_off()** can be used to skip particle splitting, for debugging purpose.

Kernel functions

krln()

The present kernel functions adopted by SPheRIO are based on cubic spline. They can be found along with a few discussions in Eq.(39) in "A pedagogical tool using SPH to model fluid flow past a system of cylinders", by Brain Schlatter, Master's thesis, Oregon State University, (1999).

Equation of state

The subroutines in this block deal with EOS . See Section III for more details.

It contains the following subroutines: **bdy0read(), bdyread(), digread(), eosph(), eosph_original(), eosph_pion1(), eosph_pion2(), eosph_pion(), eosph_pion_bag1(),**

[5] This was carried out in another implementation of SPheRIO by Ph. Mote and T. Kodama.

eosph_pion_bag2(), eosph_strangeness(), eosphdbg(), eosphtrials(), esxread(), geteosesx(), geteosmix(), getesx(), gettmunew(), ingues(), mixdphs(), mixread(), nxpd(), pdtable(), phsjdg1(), phsjdg2(), phsjdg1_original(), phsjdg1dbg(), phsjdg2dbg(), phsjdgtrials(), rftzer(), readstb(), readtb(), thintp(), thrread(), tmevol(), tmintp(), tmuread(), usrfun()

Several sets of pre-defined EOS are at disposal. They can be chosen through the setting "set *ieos an_eos_number*", where *ieos* is the keyword for the EOS setting, and *an_eos_number* is the corresponding EOS index. The first set of EOS describes matter on the hadronic side, by means of a hadronic resonance model with finite volume corrections, and MIT bag model for quark gluon plasma (QGP) phase. The main part of observed resonances in Particle Data Tables are included in the hadronic phase. Use "set *ieos* 01" to utilize this EOS . The second set of EOS is almost the same as the first set of EOS , except that local strangeness neutrality is assumed to find the strangeness chemical potential. Use "set *ieos* 02" to activate this EOS . By choosing "set *ieos* 0", one selects a set of analytic EOS with uses massless pion gas for the hadronic phase and the MIT bag model for QGP phase. The option "set *ieos* 04" uses the EOS of massless pions (π^+, π^-, π^0) for debugging purposes.

The option "set *ieos* 07" activates a set of EOS inspired in lattice QCD[6]. The EOS defined by "*setieos* 08" is essentially the same as in "set *ieos* 07", except that local strangeness neutrality is assumed. The options *an_eos_number* 07 and 08 are still under development and will be available soon.

To supply a user-defined EOS , override "virtual" functions **eosph_override()**, **phsjdg1_override()** and **phsjdg2_override()** and apply "set *ieos* 99". Therefore code will make use of the above functions in place of the original **eosph()**, **phsjdg1()** and **phsjdg2()**, consult Section III for more details.

Pure numerical recipes

The block contains those subroutines each of which has its own independent computational algorithms and usage, while they are employed as "black box" by various subroutines from different blocks of SPheRIO.

findph(), findth(), gammln(), gasdev(), indexx(), itrp2d(), locate1(), locate(), mnewt(), mnewtgetmu(), newton(), phif(), poidev(), polint11(), polint(), ran1(), ran3(), rtflsp1(), rtflsp2(), rtnewt(), velequ()

In the coming Section IV, the subroutines will be discussed in more detail.

[6] Y. Hama, R.P.G. Andrade, F. Grassi, O. Socolowski Jr., T. Kodama, B. Tavares and S.S. Padula, Nucl. Phys. A744, 169 (2006)

II. THE OPTION FILE INTERFACE

The option file ***.optns** serves as an interface to pass parameters to the code at run-time. For historical reasons, SPheRIO adopted the option file of NEXUS and went ahead to add to it more features of its own.

In the following, all the keywords provided by SPheRIO are listed [7].

The elements of the list below show examples of MACRO definitions of centrality corresponding Au+Au collisions at RHIC which can be further used to define the centrality windows customly adopted by PHOBOS or STAR collaborations. For illustrating the usage of these MACRO definitions, let us take a look at them. "#define bim06 3.64" defines a MACRO bim06 which value is set to 3.64, therefore, by "set *bminim* 0.0" (i.e., $b_{min} = 0.0$) and "set *bmaxim* bim06 (i.e., $b_{max} = 3.64$), one sets the impact parameters of the collision at the first PHOBOS centrality window: 0% - 6%. The code has been written in such a way that it will automatically find and utilize the correct rescaling factor (to adjust the pseudo-rapidity distribution of all charged particles), as well as the freeze-out temperatures, using linear interpolation. Therefore, for selecting the desired centrality window, the procedure is then to simply un-comment one of the lines in the option file below. For more information read comment of *ibugflg*.

#define bim03 2.55	#define bim05 3.32	#define bim06 3.64
#define bim10 4.73	#define bim15 5.81	#define bim20 6.68
#define bim25 7.46	#define bim30 8.17	#define bim35 8.83
#define bim40 9.42	#define bim45 10.00	#define bim50 10.55
#define bim60 11.57	#define bim70 12.48	#define bim80 13.37
#define bim92 14.57		

set *laproj* 79 set *maproj* 197 set *latarg* 79 set *matarg* 197

!!above statement set, from left to right, number of projectile protons to 79, number of projectile nucleons to 197, number of target protons to 79 and number of target nucleons to 197. Therefore it literally reads, Au-Au collisions.

set *ecms* 200.0

!!it sets the center-of-masscollision energy at 200A GeV

set *nfreeze* 10

!!it sets the number of MCs freeze-out procedures to 10, corresponding to each full hydrodynamic evolution.

set *tempoico* 1.0

!!initial time τ of the IC when hydro takens over

set *nxico* 25 set *nyico* 25 set *nzico* 25

!!it sets the number of lattice sites in x, y and η direction respectively

set *xminico* -8 set *xmaxico* 8 set *yminico* -8 set *ymaxico* 8 set *zminico* -6 set *zmaxico* 6

[7] Here some features are adopted or inspired by NEXUS/EPOS, credits are given to Klaus Werner and co-authors

!!the initial system size in question, e.g. *xminico* means the low boundary in x direction, *zmaxico* means the high boundary in η direction

fname *ico* ../optns/auau-epos-pj1.ico
    !!ico file to read as the input IC

switch *mprcsion* off
    !!run the code with the highest possible precision but very very slow

set *hxy* 1.0 set *het* 1.0
    !!set *hxy* and *het*, the transverse and longitudinal sizes of SPH particles

set *dtau* 0.25
    !!*dtau* = time step interval

set *ibugflg* 20
    !!debugging/run-time mode switcher, ibugflg has the highest priority, it may override any other parameters

!!00 - normal mode, optns file determines all the parameters, inital time, thermal/chemical freeze-out temperatures and etc
!!01 - run the code with original EOS without strangeness and corrected equation of motion
!!02 - run the code with new EOS with strangeness and corrected equation of motion
!!03 - reserved
!!07 - reserved
!!08 - run the code with EOS of massless pions, the EOM will be solved only on the x-y plane, and using the scaling solution for the η direction, one may also use the switch "skip *icotable*" in the option file, since one must provide a 2-dimensional IC
!!09 - run the code with EOS of massless pions with bag model containing a first order phase transition, using the Gaussian-like IC
!!10 - run the code with EOS of massless pions with bag model containing a first order phase transition, using NEXUS IC
!!11 - run the code with new EOS with strangeness and using the Gaussian-like IC
!!12 - run the code with new EOS with strangeness and using the NEXUS IC, it outputs some informations on fluid evolution
!!13 - run the code with new EOS with strangeness and using the NEXUS IC, it outputs more informations on fluid evolution
!!15 - run the code with EOS of massless pions, the EOM will be solved only in two dimensions
!!20 - run the code with new EOS with strangeness and using the NEXUS IC, the thermal/chemical freeze-out temperatures are automatically chosen by the code, according to the impact parameters
!!21 - the same as 20, except that it is used to calculate v2 and the yields of some particles are multiplied by a factor of 10
!!30 - run the code with original EOS without strangeness and original equation of motion and core fuction

set $iprtflg$ 9

> !!$iprtflg$ determines the debugging message output level, bigger number implies more detailed info

set *tempqmout* 0.145
!!freezeout temperature for chemical freezeout

set $tempfzout$ 0.135

> !!it sets the temperature of thermal freeze-out, this value may be overridden by $ibugflg$

set $tempfinal$ 0.090

> !!it sets the lowest temperature the fluid can reach, this value is essentially determined by the boundary of EOS table

switch *emcontinua* off

> !!run the code using thermal freeze-out scenario, this switcher has been modified from the earlier version to match the style of the rest of the program

!switch *emcontinua* on

> !!run the code using continuous emission scenario

set *kappa* 0.30

> !!parameter *kappa* determines how easily a particle can escape from the fluid, see eq.(123) of Yogiro's paper: "Topics on hydrodynamic model of nucleus-nucleus collisions", Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264].

set *idecomode* 01

> !!01 - isotropic original formulae for continous emission
> !!02 - anisotropic momentum dependent formulae, momentum dependent formulae, it determines how hadrons emit from the frozen-out SPH particles the first choice "isotropic" uses regular isotropic formula, which takes into account fluid velocity, momentum of emitted particle, isentropic surface and free Fermi/Boson distribution, see eq.(111) of Yogiro's paper: "Topics on hydrodynamic model of nucleus-nucleus collisions", Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264]. The second choice takes into account that even the free Fermi/Boson distribution is not isotropic, it can be expressed in terms of interaction distribution as $f_{free} = CP/(1-P)f_{int}$, where P is the probability of emission and it is anisotropic, and C is a constant regarding normalization, for details see comments of the code

set *itrinimode* 00

> !!this swtcher provides user with some predefined IC to choose from, it has four options, pure NEXUS IC, NEXUS plus some initial transverse velocity using two different formulae and Mach Cone IC
>
> > !!00 - pure NeXus initial transverse boost
> > !!09 - NeXus+tanh() initial transverse boost
> > !!11 - a way of transverse boost using rapfactor
> > !!15 - Mach Cone IC

switch *sphsplit* off

  !!turn off SPH particle splitting

switch *freezeout* on

  !!to evoke the freeze-out process or simply skip it

!read *fzoutTable* - fname *fzout* ../optns/media-auau-pj1.fzo.data
!write *fzoutTable*

  !!to assign the filename, *fzout*, to read/write from which contains the freeze-out suface information in terms of SPH particles

switch *quimfz* on

  !!employ chemical freezeout, when chemical freeze-out is on, the code further provides four type of chemical freeze-out scenarios, namely

set *iquimmode* 06

  !!01 - chemical and thermal freeze out of hyperon at hadron phase boundary
  !!02 - chemical and thermal freeze out of hyperon at qgp phase boundary
  !!03 - chemical and thermal freeze out of hyperon at temperature tempqmfzout
  !!04 - chemical of hyperon at tempqmfzout, thermal freeze out at tempfzout
  !!05 - scenario specifically for omega
  !!06 - the same scenario as 04 but for all particles

switch *ihbt* off

  !!when this switcher is on, it calculates HBT radius

set *ieos* 02

  !!this parameter sets which EOS is to be used, it can be overridden by *ibugflg*
    !!01 - MIT bag + hadron resonance without strangeness
    !!02 - MIT bag + hadron resonance with strangeness
    !!03 - MIT bag + massless pion
    !!04 - massless pion gas

set *ihydromode* 00

  !!00 - original mode, SPH decouples at *tempfzout*
  !!01 - violent mode, SPH decouples at freeze-out surface
  !!This parameter gives options to run the code using two different EOM. In the first case, when it sets to zero the SPH particles continuous interacting with others even after freeze-out in the second case, when it sets to 1, frozen-out SPH particles are taken away from EOM, they remain their velocities when they decouple from the fluid, although they are counted when calculating the probability of particle emission

set *itrobmode* 00

  !!this parameter controls the level of outputs of information on the evolution of the system the output file contains velocity, entropy density, temperature and other thermodynamic quantities of the fluid and SPH particles
    !!00 - no output
    !!01 - standard output at $\tau = 1, 3, 5, 10$, output on x and y axis
    !!02 - standard output at $\tau = 1, 2, 3, ...$, output on x and y axis

!!08 - specific output for massless pion gas, output and raw output on transverse plane, corresponding to ibugflg 08
!!09 - specific output at $\tau = 1, 4, 7, 10$, raw output on x, y and r direction, corresponding to ibugflg 09
!!12 - specific output at $\tau = 1, 3, 5, 10$, output on transverse plane, corresponding to ibugflg 12
!!13 - specific output at $\tau = 1, 2, 3, ...$, output on x and y axis and raw output on individual SPH particle, corresponding to ibugflg 13
!!14 - specific output at $\tau = 1, 2, 3, ...$, output on transverse plane
!!15 - specific output for massless pion gas, corresponding to $ibugflg$ 15

set *idimension* 3
!!3D calculation or 2D limit with specified IC

set *iviscosity* 01
!!00 - ideal hydro, algorithm 1: non-linear equation
!!01 - ideal hydro, algorithm 2: iteration
!!02 - bulk viscosity
!!03 - shear viscosity and bulk viscosity

III. CONVENTIONS OF EOS DATAFILE

The EOS data files are made more complicated than it would have to be. The reason is that SPheRIO needs to know in which phase a certain SPH particle is located when dealing with the freeze-out. (For example, in QGP phase the SPH particles do not suffer freeze-out in "Continuous Emission" scenario)

The data files concerning EOS infomation are: **bdy-hrg0.dat**, **bdy-qgp0.dat**, **bdy-hrg.dat**, **bdy-qgp.dat**, **eos-eq.dat**, **eos-er.dat**, **eos-sq.dat**, **eos-sr.dat**, **mix-ee.dat**, **mix-tp.dat**, **stabl1.dat**, **stabl2.dat** and **stabl3.dat**.

The **bdy-qgp.dat** and **bdy-hrg.dat** contain the information about boundaries between QGP phase/mixed phase and hadronic phase/mixed phase. Both of them are one-dimensional. I will take **bdy-qgp** as an example, because the data structures in both files are very similar.
– The first line contains the number of columns, e.g., 200,
– The second line is just a comment line, however, please maintain it unchanged, since the subroutine to read the file is not very smart.
– After that there are successive 200 lines, each of them contains 15 numbers, which are
* temperature (xtm) – the only variable since it is one dimensional
* baryon chemical potential ($xmub$)
* strangeness chemical potential ($xmus$)
* energy density on the QGP side ($eedstqgp$)
* entropy density on the QGP side ($tpdstqgp$)
* baryon density on the QGP side ($bbdstqgp$)
* strangeness density on the QGP side ($ssdstqgp$)
* strangeness fraction (= strangeness density / baryon density) on the QGP side ($xfsqgp$)
* pressure on the QGP side ($ppdstqgp$)
* energy density on the hadronic side ($eedsthrg$)
* entropy density on the hadronic side ($tpdsthrg$)
* baryon density on the hadronic side ($bbdsthrg$)
* strangeness density on the hadronic side ($ssdsthrg$)
* strangeness fraction (= strangeness density / baryon density) on the hadronic side ($xfshrg$)
* pressure on the hadronic side ($ppdsthrg$=$ppdstqgp$)
If one draws the phase boundary in terms of temperature vs. baryon chemical potential, there are two curves corresponding to the boundary of the QGP phase and hadronic phase. This is the reason why two files are needed to store the information. In the case when strangeness is not considered, the data in the two files are the same.

bdy-hrg0.dat and **bdy-qgp0.dat** store the information at T=0 of hadronic phase and the QGP phase. Pratically, due to numerical issues (although one can solve the equations strictly at T=0, at very small T it turned out to be difficult) the temperature was taken as 0.01GeV. Take $bdy - qgp0$ as an example.
– The first line contains the number of columns, e.g., 200.
– The second line is just a comment line, however, please keep it unchanged, since the subroutine to read the file is not very smart.
– After there are other successive 200 lines, each of them containing 7 numbers. These are
* the temperature (xtm) – the only variable since it is one dimensional
* baryon chemical potential ($xmub$)

* strangeness chemical potential (*xmus*)
* energy density (*eedstqgp*)
* entropy density (*tpdstqgp*)
* baryon density (*bbdstqgp*)
* pressure (*ppdstqgp*)
In the QGP phase, the initial baryon density is not necessarily zero, so I start with a number a little smaller than the that corresponding to the deconfinement transition, i.e., 0.001fm^{-3}.

eos-eq.dat, **eos-er.dat**, **eos-sq.dat**, **eos-sr.dat**, **mix-ee.dat** and **mix-tp.dat** are the files store the information in QGP and hadronic phases in terms of energy density/baryon density or entropy density/baryon density.
– Taking **eos-eq.dat**, for example, **e** (of eq) stands for energy density, and **q** stands for QGP phase, so this file contains the EOS information of QPG phase.
– The first line contains the numbers of two elements of the two-dimensional array, e.g,. 150 (the loops of baryon density) 150 (the loops of energy density)
– After that was successive 150×150 lines, each of them contains 7 numbers, they are
* energy density (*edeq*) – the inside (second) loop index, according to the file, it goes from 1.6GeV (phase boundary at $rhob = 0$) to 21GeV
* entropy density (*sdeq*)
* baryon density (*rhobeq*) – the outside (first) loop index, according to the file, it goes from 0.00001fm^{-3} to 2.5fm^{-3}; note that the pace was not equally divided
* pressure (*ppeq*)
* temperature (*tteq*)
* baryon chemical potential (*mubeq*)
* strangeness chemical potential (*museq*)

For the file **eos-sq.dat**, the data are orgnized in the same order, except one should understand it as
* energy density (*edsq*)
* entropy density (*sdsq*)– the inside (second) loop index
* baryon density (*rhobsq*) – the outside (first) loop index
* pressure (*ppsq*)
* temperature (*ttsq*)
* baryon chemical potential (*mubsq*)
* strangeness chemical potential (*mussq*)

The files **stabl1.dat**, **stabl2.dat** and **stabl3.dat** concern the particle emission only so, in principle, they do not need to be modified when one does not consider strangeness or other conserved charges. Note that some data in the tables overlap with one another, the names of the variables are adopted from the subroutine **readtb()**. The conventions are as follow,
* *temp* – temperature
* *chemb* – baryonic chemical potential
* *utb* – chemical potential of proton divided by temperature
* *utab* – chemical potential of proton-bar divided by temperature
* *denom* – the correction factor on the denominate, when involving finite volume correction. See formulae in "Topics on hydrodynamic model of nucleus-nucleus collisions", Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264]. It is worth noting that finite volume correction eq.(28-29) of this article are used, that's why $utb! = -utab$
* *ut* – chemical potential divided by temperature

* *mt* – mass divided by temperature
* *xintg* – baryonic number density divided by temperature cube This table is used to save time in the calculation, note *xintg* is a quantity that stays constant when ut and mt are given

xx_T - temperature xx_ub - baryonic chemical potential xx_P - pressure.
The dimension of the table are determined by $ntb1x$ and $ntb1y$, volume corrections were considered.

In the new version of SPheRIO I added data of chemical potential and modified some structure of the last three tables as well as removed some overlapped data. But the basic idea remains the same.

IV. FUNCTION AND ALGORITHM OF SUBROUTINES

In the following, we will discuss the function and algorithm of each subroutine. Some efforts are made to adopt the comment style of *"Numerical Recipes: The Art of Scientific computing 3rd Edition" Cambridge University Press, ISBN-13 978-0-511-33555-6/ISBN-13 978-0-521-88068-8.*, i.e., instead of telling trivial detials about what each statement line is doing, we try to focus on how the subroutines fit and serve in the whole program and why certain algorithm is employed.

Some in-line comments can be found in SPheRIO's source files and updates are logged on top of the subroutines.

Initialization

rfactor_*(), config(), config_pion(), latt(), loadeos(), machconeini(), modeprint(), newvel(), rfactor(), rfactorprint(), rfactorset(), seteosmode(), set_flagce(), set_flagxn(), setin(), setin_kodama(), setin_pion(), setkappa(), setlog(), settautrans(), settempsph(), settempsphdenovo(), settrinimode()

config() divides the space-time into discrete lattices. The range of space-time under consideration can be readjusted in accordance with the value of $bugflg$, since $bugflg$ has a higher priority than the *optns* settings. However, the total number of initial SPH particles is determined by nxico, nyico and nzico. These numbers are adopted by SPheRIO as parameters. In case one wants to change them, edit NEXUS' file *nexus.incico*. **latt()** is called at the end of subroutine. The program structure between **config(), latt()** and **setin()** is to be improved.

config_pion() serves in the same way as **config()**, except that it divides space-time with cylindrical symmetry and it implements the function of **latt()** and **setin()** within itself. For the time being, the subroutine is used in the case of massless pions, however it is easy to be modified and applied to different situations.

latt() has within itself a loop enumerating all the lattice sites, this is when **setin()** or **setin_kodama()** or **setin_pion** is invoked. These subroutines calculate and assign dynamic quantities (four velocity) and thermodynamic quantities (entropy density, energy density, volumn and etc) to the corresponding SPH particle sitting on the very lattice site. Then the total number of SPH particles is estimated and passed to **Nsp(1)**. If one calcuates "Mach Cone', the subroutine **machconeini()** is invoked to distribute an extra SPH particle on top of others.

setin() calculates dynamic quantities (four velocity) and thermodynamic quantities (entropy density, energy density, volumn and etc) at a given lattice site using the NEXUS initial conditions. Firstly, **newvel()** is called to solve the four velocity by the equation $T^{\mu}_{\nu}u^{\nu} = \epsilon u^{\mu}$. The resulting velocity is used to calculate flavor charge **IcoF** and invariant energy **IcoE**. Then the rescaling **factor()** is applied. Baryon density, electric charge density, strangeness density are evaluated. If an SPH particle stays beyond the boundary of EOS (its phase is indeterminable), it is thrown away. If the temperature of SPH particle is lower than the freeze-out temperature T_f, but higher than the EOS table limit, its properties will be stored in the "*ic*" arrays for future use. This particle is then excluded from EOM. Some initial transverse boost profile is applied according to the value of "*itrans*".

rfactor() is an energy rescaling factor as a function of spatial coordinate eta. The function calls one of different implementations **rfacor_*()** according to the value of index variable $irft$. When $irft$ does not match any value in a pre-defined list, an error echos and the program halts.

rfactor_*() implements the rescaling factor. For instance, **rfactor_bug20_t1a0_mus_js1()** is used when $bugflg = 20$, $\tau_0 = 1.0$, EOS with strangeness, in the first STAR centrality window 0 - 5%. **rfactor_unit()** is a trivial case where the rescaling factor is a constant, 1.

rfactorset() is executed before that **rfactor()** is invoked by **setin()**. Its goal is to assign a correct value to $irft$ according to collision energy, impact parameters, τ_0, freeze-out scenarios and $ibugflg$. What we intended is to make the choice of rfactor as automatic as possible, obviously, it didn't work out very well. **rfactorset()** and **rfactor()** look fat and they are difficult to maintain.

loadeos() loads EOM tables into the code according to *ieosmode*.

seteosmode() is executed before **loadeos()**, it determines the value of *ieosmode* by $bugflg$.

machconeini() creates an heavy SPH particle with a different velocity and entropy on top of the IC generated by **setin()**. It is used to creat an initial conditions for the study of "Mach Cone".

modeprint() prints the de facto value of all the major parameters in the initilization block into the main surveillance file *ztr.data*.

newvel() solves the four velocity by the equation $T^{\mu}_{\nu}u^{\nu} = \epsilon u^{\mu}$.

rfactorprint() prints out the function of rescaling factor vs. eta, for debugging purpose.

set_flagce() and **set_flagxn()** set initial "$0'$ to freeze-out flags $iflgce$ and $iflgxn$.

setkappa() sets the value of "*kappa*", the parameter in Eq.(123) of "Topics on hydrodynamic model of nucleus-nucleus collisions" Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264].

setlog() resets all the log files.

settautrans() sets τ_0, the initial time when NEXUS hands down the initial conditions, and SPheRIO starts the hydro evolution.

settempsph() sets thermal/chemical freeze-out temperatures accroding to optns and freeze-out scenarios.

settempsphdenovo() automatically readjusts the thermal/chemical freeze-out temperatures according to the value of bugflg.

settrinimode() determines the initial transverse boost velocity and new EOM scenario where SPH particles are/not taken away from fluid when they are frozen out.

Hydrodynamic evolution

dqfn(), dqfn_xy(), findbt(), findbt_xy(), firststep(), fldobsrv(), fldobsrv_fin(), outp_raw(), rung(), solvediff(), tranisotropy(), trobsrv(), trobsrv_old(), trobsrv_pion()

solvediff() solves the hydrodynamic equation Eq.(100) of "Topics on hydrodynamic model of nucleus-nucleus collisions" Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264]. It employs 2^{nd} order Adams-Moulton method to solve the second order ordinary differential equation. To obtain velocity from momentum, a nonlinear equation is made use of (In fact, this is not necessary since one may derive the EOM in terms of uT). In the nonlinear equation, the variable chosen to be solve is $\beta = ln(\sqrt{1+u_T^2}+u_T)$. β goes to infinite when velocity approaches the speed of light, hence the choice of beta ensures the precision at high velocity region. Since ν_i cancels on both sides of EOM, for convenience, the canonical momentum q is defined as pi_i/ν_i. The subroutine also does maintenance of SPH particle data for itself and other blocks such as freeze-out, particle splitting, link-list and etc. In the follows,, we explain how the subroutine works. First, **firststep()** is invoked to do some initialization. *ihdfull* and *ihdmode* are checked here and from time to time later on to implement the new EOM. The new EOS works by making a link-list excluding those frozen-out particles. "ihdmode" determines whether new EOM optns is turned on, *ihdfull* indicates at run-time whether one should build the link-list with/not those frozen-out particles. The information on SPH particles of two time steps in the past is stored in arrays with "*oflg*" being -2 and -1. This is not because we make use of a 2nd method to solve EOM but that thermal freeze-out employs three point interpolation. Particle decay is treated by **dectrl()** or **dectrl_off()** before the time evolution takes place. The Adams-Moulton method is implemented in two steps corresponding to two instances τ and $\tau + d\tau$. In the first step, "*oflg*" is set to 1, **mklt()**, **prcn()**, **fins()**, **bsqd()** are invoked to update the link-list, and compute the thermodynamic quantities of each SPH particle. **dqfn()** is used to evaluate the r.h.s of Eq.(100). **rung()** calcuates dynamic quantities. In the second step, the code repeats itself except "*oflg*" is set to 2 and the time τ evolves. Then the SPH particles are sorted using **indexx()** in terms of temperature in decreasing order. Since each SPH particle in principle will be assigned to a different index number, the father-son relationship between SPH particles (due to particle splitting) has to be carefully taken care of. In the end, low temperature ($T < Tfin$) SPH particles are cut away from the **EOM**. Throughout the subroutine, status of **SPH** particles as well as total entropy and energy conservation are verified, and the results are directed to the surveillance file **ztr.data**.

Here is a list of EOM arrays used to store information on SPH particles during the hydro evolution. Wtih the exception of Nsp(oflg) being the total number of SPH particles in EOM, all the rest of them share the same form.

EOM_Array_Name() := EOM_Array_Name(oflg, EOM_SPH_particle_index),

where "*oflg*" is the status *flag*: -2 and -1 are used in interpolation of Cooper Frye, 0 is reserved for temporary storage like sorting, 1 and 2 are used in 2nd order Adams-Moulton method. **xx()=xsph(1) x** coordinate, **yy()=xsph(2)** y coordinate, **et()=xsph(3)** eta coordinate, **bt()=rsph(1)** beta, **ph()=rsph(2)** phi, **al()=rsph(3)** alpha, **sn()** entropy, **br()** baryon number, **sr()** strangeness charge, **qr()** electric charge, **ss()** proper entropy density, **bd()** proper baryon density, **sd()** proper strangeness density, **qd()** proper electric charge density, **mbxn()** baryon chemical potential of chemical freeze-out, **msxn()** strangeness chemical potential of chemical freeze-out, **tempxn()** temperature of chemical freeze-out, **uxn()** contracovariant four velocity corresponding to $ufcc$ not $ufrs$.

firststep() does the initialization. It sorts SPH particles in terms of temperature and prints out run-time value of some quantities in *ztr.data*.

dqfn() evaluates the r.h.s of Eq.(100), with **dqfn_xy()** being the 2D implementation of **dqfn()**.

rung() implements the 2^{nd} order Adams-Moulton method (though it is named after Runge-Kutta by a historical mistake). The subroutine calculates the dynamic quantities of SPH particles. It first computes the momentum q, and add to it dq obtained by **dqfn()**. Then **rtflsp1()** is employed to solve the equation of beta findbt(). With beta obtained, other dynamic quantities like vx, vy, ve, $alpha$, phi can be acquired easily. Here is a list of definitions of the variables envolved. $rsph(1) = \beta = \ln(\sqrt{1+u_T^2} + u_T)$, $rsph(2) = \phi = atan(vy/vx)$, $rsph(3) = \alpha = 1/2\ln((1+\tau v_e)/(1-\tau v_e))$. It is easy to verify that four velocity can be expressed in terms of $rsph$ arrays: $u_0 = \cosh(rsph(3) - xsph(3))\cosh(rsph(1))$, $u_e = \sinh(rsph(3) - xsph(3))\cosh(rsph(1))/\tau$, $u_x = \cos(rsph(2))\sinh(rsph(1))$, $u_y = \sin(rsph(2))\ sinh(rsph(1))$.

findbt() is the equation of beta, with **findbt_xy()** being its 2D version.

fldobsrv() and **fldobsrv_fin()** print out longitudinal velocity (alpha) distribution of SPH particles at the beginning and end of hydro evolution.

outp_raw() prints out transverse spatial distribution of SPH particles.

tranisotropy() computes momentum anisotropy as a function of time.

trobsrv(),**trobsrv_old()**, **trobsrv_pion()** print out transverse distribution of thermodynamic quantities. The difference between the three subroutines is how to evaluate the quantity at a given spatial point.

Freeze-out

checkce(), checknu(), checkxn(), flintp(), lssf(), normal(), normal_ce(), probce(), probce_gen(), recordxn(), recordxn_inspector()

lssf() is the meat and potatoes of freeze-out block. **lssf()** treats Cooper Frye freeze-out scenario within itself, it calls **checkce()** to handle continuous emission, **checkxn()** and **recordxn()** to observe chemical freeze-out. Here is how it works. **checkxn()** and **checkce()** are called first off. The two subroutines check SPH particles one by one to see if they meet the criteria of freeze-out. If so, the freeze-out flags ($iflgce$ and $iflgxn$) of those particles switch from "0" to "1" to signal for further treatment. After that, it is a check point of' $ihdmode$ and $ihdfull$ for the new EOM. Then it comes to two successive blocks to handle continuous emission and Cooper Frye. Since the structures of the two blocks are very similar, we will only focus on the first block of continuous emission. First, the freeze-out flag $iflgce$ switches from "1" to "2" to indicate that SPH particle has been taken care of. Information on dynamic quantities (coordinate, velocity) and thermodynamic quantities (entropy, baryon number and their densities, as well as temperature) on freeze-out surface is copied from EOM arrays to freeze-out arrays. The copies are made first for continuous emission then for chemical freeze-out, if the operations are legitimate. The covariant normal vector of freeze-out surface is evaluated by **normal_ce()** and stored, as well as covariant four vector $sfrs$ (It is the term in the summation on the r.h.s of Eq.(111) excluding contravovariant momentum p and statistical distribution f, we will refer to it as normal flow thereafter) . The difference between the block of kinematic freeze-out and continuous emission is that it employs three point to calculate the quantities on freeze-out surface. To meet the kinematic freeze-out criteria, the SPH particle has to stay below freeze-out temperature Tf for two successive time steps. The subroutine comes to end with another check point of $ihdmode$ and $ihdfull$ for the new EOM.

Here is a list of freeze-out arrays used to store information on freeze-out surface. All of them share similar form.

$FO\_Array\_Name(*) := FO\_Array\_Name(*, FO\_SPH\_particle\_index)$.

$xfrs(0)$ τ, $xfrs(1)$ x coordinate, $xfrs(2)$ y coordinate, $xfrs(3)$ eta coordinate, $rfrs(1)$ β, $rfrs(2)$ phi, $rfrs(3)$ $alpha$, $ssfz()$ entropy density, $snfz()$ entropy, $bnfz()$ baryon number, $mfrs()$ baryon chemical potential, $msfrs()$ strangeness chemical potential, $edfz()$ energy *density*, $bbfz()$ baryon density, $tempfz()$ temperature, $nrfz(0-3)$ normal vector, $ppfz()$ probability of escape via continuous emission, $sfrs(0-3)$ normal flow, $mfrsxn()$ baryon chemical potential of chemical freeze-out, $msfrsxn()$ strangeness chemical potential of chemical freeze-out, $tempfrsxn()$ temperature of chemical freeze-out, $ufrsxn()$ uxn of chemical freeze-out.

checkce() invokes **probce()** to calculate the escape probability of SPH particle and make use of MC to decide whether an SPH particle freezes-out. The new EOM are also implemented in this subroutine. When calculation the escape probability, all particles should be taken into account, thus $ihdfull$ is set to 1, after that, $ihdfull$ is switched back to 0.

probce() evaluates the escape probability of a given SPH particle. It first calculates the percentage of escaping particles pp using a momentum independent approximation Eq.(123). Its first order derivative measures the rate of SPH particle freezing. Taking into consideration that we only select from the free SPH particles in EOM, a factor $1/(1-pp)$ is included.

probce_gen() returns the escape probability of a given coordinate, in stead of a SPH particle, in this sense this subroutine has more general usage.

checkxn() verifies if a SPH particle suffers chemical freeze-out. If it does, the flag $iflgxn$ is set to 1 and **recordxn()** is invoked to store the information. There are several senarios of chemical freeze-out, chemical freeze-out may take place on phase boundaries or at a fixed temperatures.

recordxn() makes copy of quantities of chemical freeze-out particles into the arrays with keyword xn. It uses two point interpolation if chemical freeze-out takes place on phase boundaries, and three point interpolation is employed if chemical freeze-out temperature is adopted. The latter case uses the same algorithm as **flintp()**. $iflgxn$ is set to 2.

normal_ce() evaluates the covariant normal vector of freeze-out surface, it is obtained by evaluting the gradient of isentropic surface. It is straightforward to verify the formulae used in the code, we only note here that derivative of $\gamma = \cosh(al - eta)\cosh(bt)$ on the denominate results in an extra term which presents itself at the end of the subroutine.

normal() is similar to **normal_ce()** except that it calculates the normal vector of isotherm.

flintp() does a three point interpolation to extract the value of thermodynamic quantities at the thermal freeze-out temperature.

recordxn_inspector() is a surveillance script check whether $recordxn()$ gives consistent results.

Hadron emission

checkodd(), checkomg(), checkv2(), density(), evgen(), lorenz(), pdmax(), peso(), pesotest(), pesotrials(), prodis(), rot(), setevg(), setfac(), setnex(), settot(), setus(), weigpr()

setevg() uses statistical distribution function to estimate emission number of each species of hadrons, energy conservation is not explicitly taken into account here. The subroutine starts a loop to enumerate all the SPH particles, and evaluates the emission number **wg()** for each species using **peso()** or **weigpr()** times a volume correction factor obtained by **setfac()**. Then the emission numbers are summed up with respect to all SPH particles. If a frozen-out SPH particle is not in the hadronic phase, a correction factor is applied. The results are stored in **ntot()** array.

evgen() employs random number generator to produce the hadrons, and the data are sent back to NEXUS. One thing worth mentioning is that when HBT radius is calculated, the **evgen()** is skipped to save CPU time, since this part of code will repeatedly be executed many times when nfreeze is big. HBT radius has its own method to deal with hadron emission. In the first place, **settot()** is invoked to transverse **ntot()** into an integer taking into account statistical deviations. p**rodis()** is used to calculate the emission probability of a hadron with a certain momentum, where **rot()** and **lorenz()** help to transfer the momentum into laboratory frame. If the hadron survives the random draw **ran1()**, it is accepted and **setnex()** passes its information to NEXUS.

setnex() sends the data of emitted hadrons to NEXUS through common arrays.

checkodd(), **checkomg()**, **checkv2()** check whether a hadron satisfies certain properties. **checkodd()** checks if the hadron is a strange particle, **checkomg()** checks if it is Ω and **checkv2()** checks if the particle is proton or anti-proton.

rot() calculates the momentum of hadron in proper frame, but along the directions of axes in laboratory frame, which eases the job of **lorenz()**

lorenz() carries out the Lorentz transformation from proper frame to laboratory frame of a hadron emitted from a given SPH particle. The input four momentum is **ppr()**, the output variable is **plb()**.

prodis() evaluates the emission probability of a hadron. Anisotropic formulae are adopted in the present version.

pdmax() evalutes the maximum of probability of hadrons emitting from a given SPH particle. They do their work in a rough way, the present algorithm is to be improved.

peso() estimates the emitting hadron number using approximating statistical distribution. The momentum integral is carried out using extended Simpson's rule (see "Numerical Recipes").

pesotest() and **pesotrial()** are debugging subroutines.

density() calculates the density of hadron by its chemical potential mass and temperature using Gaussian Quadratures (See "Numerical Recipes")

setfac() evalutes the volume correction factor of emission number of hadrons.

weigpr() evaluates the emitting hadron number using Eq.(111), where a part of r.h.s. of equation is already stored in the array **sfrs()** and made use of. The momentum integral is carried out by extended Simpson's rule.

HBT radius

corr(), evghbt(), fndk(), raplist()

Eq.(120) and Eq.(122) of the paper "Topics on hydrodynamic model of nucleus-nucleus collisions", Braz. J. Phys. **35**, 24 (2005) [arXiv: hep-ph/0407264] are used to calculate the coefficient C2. A link-list method in rapidity space is implemented in the subroutine **raplist()**.

Link-list method

bsqd(), bsqd_xy(), fins()fins_xy(), finsb_gen(), finsb_gen_xy(), mklt(), prcn()

The method can be found in Numerical Recipe book. The goal of this block is provide a efficient way to calculate any dynamic or thermodynamic quantity at any given point.

subroutine **mklt()** is to build the link-list, **prcn()** is to pick up nearby SPH particles using the link-list, while **finsb_gen()** among others is to calculate the quantities at a given point.

Particle splitting

decay(), dectrl(), dectrl_off()

Particle splitting is introduced due to the fact that SPH particle might not be evenly distributed in space due to the hydro evolution. Note that the kernel function we employed expands $4hxy$. If in a cube of $(4hxy)^2(4het)$, the number of SPH particles is smaller than 64 (this estimation is not exact though), the smoothness of SPH formulation is questionable, especially when some SPH particles carry large entropy. (It is arguable whether particle splitting is the right way to heal the problem. An alternative option is that every SPH carries the same amount of conserved quantity eg. entropy and there is no decay. Initially SPH particles are not uniformly distributed, that the number density of SPH particles is proportional to entropy density. In this case, sometimes a very big SPH particle number is needed to achieve desired resolution.) **dectrl()** determines when SPH particle splitting takes place, **decay()** implements the splitting, **dectrl_off()** is a subroutine to skip particle splitting for debugging purpose.

Kernel functions

krln()

The present kernel functions adopted by SPheRIO are based on cubic spline. They can be found along with a few discussions in Eq.(39) in the article "A pedagogical tool using SPH to model fluid flow past a system of cylinders" by Brain Schlatter.

Equation of state

bdy0read(), bdyread(), digread(), eosph(), eosph_original(), eosph_pion1(), eosph_pion2(), eosph_pion(), eosph_pion_bag1(), eosph_pion_bag2(), eosph_strangeness(), eosphdbg(), eosphtrials(), esxread(), geteosesx(), geteosmix(), getesx(), gettmunew(), ingues(), mixdphs(), mixread(), nxpd(), pdtable(), phsjdg1(), phsjdg2(), phsjdg1_original(), phsjdg1dbg(), phsjdg2dbg(), phsjdgtrials(), rftzer(), readstb(), readtb(), thintp(), thrread(), tmevol(), tmintp(), tmuread(), usrfun()

See Section III.

Pure numerical recipes

findph(), findth(), gammln(), gasdev(), indexx(), itrp2d(), locate1(), locate(), mnewt(), mnewtgetmu(), newton(), phif(), poidev(), polint11(), polint(), ran1(), ran3(), rtflsp1(), rtflsp2(), rtnewt(), velequ()

There are remarks on top of each subroutine, please read corresponding chapter in Numerical Recipe book for details.

Programming style and convention

The version number of **SPheRIO** is defined in its main program, eg. PARAMETER (version='Version 200806021200BR') Each subroutine starts with a brief log on its functions and updates, it looks like the following

CCC ###################...
CCC eosph.f controls everything for **EoS** in **SPheRIO**
CCC gf: 1- energy density, 2- entropy density
CCC eos(1)= energy density...
CCC eos(2)= entropy density...
CCC eos(3)= baryon density...
CCC eos(4)= pressure...
CCC eos(5)= temperature...
CCC eos(6)= baryon chemical potential.
CCC eos(7)= strangeness chemicalpotential –¿ to incorporate strangeness
CCC ###################...
CCC last updated 06122007
CCC incorporated flag "*ieosmode*"
CCC
CCC last updated 03052007
CCC incorporated some debugging statements
CCC
CCC last updated 09122006
CCC ###################...

It is noting that a separating line CCC ####... starts with CCC and three blanks and 65 successive #s

GLOSSARY

Conventions: throughout this text, we have been using bold letters to indicate **subroutines** and italic type letters to indicate *variables*, it is adopted in following table. Note: while the following table is not fully completed, please use the SEARCH function of any text editor to locate where a specific function is evoked via the keyword "call **function_name()**", or SEARCH the name of the function, "**function_name()**", in this document for its algorithm and usage.

Name	Where it is called	What it does
config()	-	initializes the lattices
config_pion()	-	initializes the lattices
dectrl_off()	-	skips particle splitting for debugging purposes
latt()	-	configures all the lattice sites
loadeos()	-	loads the EOM tables
machconeini()	-	sets Mach Cone IC
modeprint()	-	prints parameters and other infomation
newvel()	-	redefines velocities of SPH particles
rfactor()	-	entry of longitudinal energy rescaling
rfactorprint()	-	prints information on **rfactor()**
rfactorset()	-	cofigures longitudinal energy rescaling
rfactor_*()	-	parameterizes longitudinal energy rescaling
seteosmode()	-	sets *ieos*
set_flagce()	-	sets $iflgce$
set_flagxn()	-	sets $iflgxn$
setin()	-	configures each SPH particles
setin_kodama()	-	configures each SPH particles
setin_pion()	-	configures each SPH particles
setkappa()	-	sets *kappa*
setlog()	-	sets log file
settautrans()	-	sets *tauico*
settempsph()	-	sets $tempfzout$ $tempfinal$ $tempqmout$
settempsphdenovo()	-	sets $tempfzout$ $tempfinal$ $tempqmout$
settrinimode()	-	sets $ihdfull$

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