Aspects of Experimental High-Energy Physics

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Outline

- Introduction to the Standard Model
- Accelerators and Detectors
- Data Reconstruction
- Data Analysis
- (Re-)Interpretation

Aspects of HEP – Introduction to the Standard Model

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Disclaimer

- Every physicist working in HEP should have a strong knowledge of the Standard Model, its strengths and shortcomings.
- Of course, it is impossible to obtain such knowledge in ~~one week~~. one lifetime!
- What I will try to give you in this lecture you is something akin to a treasure map.
  - It shows you the way, and highlights some features.
  - But it is by no means complete, and sometimes you will not understand it!
- I would like to thank Prof. Novaes, Prof. Gregores, and Prof. Ponton (RIP) for their contributions to this lecture.
- Finally, if you want to study the Standard Model in depth (you should!), I recommend:
  - "Standard model: An Introduction", S. F. Novaes, https://arxiv.org/abs/hep-ph/0001283
  - "Quarks & Leptons: An Introductory Course In Modern Particle Physics", F. Halzen and A. Martin. Wiley.
  - "The Standard Model and Beyond", P. Langacker. CRC Press.

The Standard Model

- Model of electromagnetic, weak and strong interactions.
- Reproduces **extremely well** the phenomenology of all observed particles.
- Based on:
  - Experimental discoveries:
    - Positron (1932), muon (1937), strange (1953–54), charm (1974), tau (1975), ...
  - Quantum Field Theory: particles are quanta of fundamental fields.
    - Quantum Mechanics + Special Relativity.
  - Invariance under tranformations that belong to symmetry groups.
    - Interaction comes as result of fundamental symmetries.
- Successful predictions:
  - Existence of neutral currents that mediate the weak interactions.
  - Mass of W and Z bosons.
  - Equal numbers of leptons and quarks in isospin doublets.
  - Existence of scalar neutral boson (Higgs boson).

THE STANDARD MODEL OF

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model is a quantum theory that summarizes our current knowledge of the physics of fundamental particles and fundamental interactions (interactions are manifested by forces and by decay rates of unstable particles).

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-2)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-2)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.05-2)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

*See the neutrino paragraph below.

Spin is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s $=1.05\times10^{-34}$ J s.

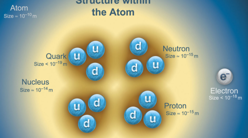
Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The energy unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$) where 1 GeV = 10^9 eV $=1.60\times10^{-10}$ joule. The mass of the proton is 0.938 GeV/c^2 = 1.67×10^{-27} kg.

Neutrinos
Neutrinos are produced in the sun, supernovae, reactors, accelerator collisions, and many other processes. Any produced neutrino can be described as one of three neutrino flavor states ν_e, ν_μ, or ν_τ, labelled by the type of charged lepton associated with its production. Each is a defined quantum mixture of the three definite-mass neutrinos ν_L, ν_M, and ν_H for which currently allowed mass ranges are shown in the table. Further exploration of the properties of neutrinos may yield powerful clues to puzzles about matter and antimatter and the evolution of stars and galaxy structures.

Matter and Antimatter
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$ but not $K^0 = d\bar{s}$) are their own antiparticles.

Structure within the Atom



If the proton and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Higgs Boson spin = 0		
Name	Mass GeV/c^2	Electric charge
H Higgs	126	0

Higgs Boson
The Higgs boson is a critical component of the Standard Model. Its discovery helps confirm the mechanism by which fundamental particles get mass.

Color Charge
Only quarks and gluons carry "strong charge" (also called "color charge") and can have strong interactions. Each quark carries three types of color charge. These charges have nothing to do with the colors of visible light. Just as electrically-charged particles interact by exchanging photons, in strong interactions, color-charged particles interact by exchanging gluons.

Quarks Confined in Mesons and Baryons
Quarks and gluons cannot be isolated – they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs. The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge.

Two types of hadrons have been observed in nature **mesons** $q\bar{q}$ and **baryons** qqq. Among the many types of baryons observed are the proton (uud), antiproton ($\bar{u}\bar{u}\bar{d}$), and neutron (udd). Quark charges add in such a way as to make the proton have charge 1 and the neutron charge 0. Among the many types of mesons are the pion π^+ ($u\bar{d}$), kaon K^- ($s\bar{u}$), and B^0 ($d\bar{b}$).

Learn more at ParticleAdventure.org

Properties of the Interactions

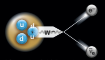
The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

Particle Processes

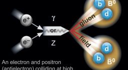
These diagrams are an artist's conception. Orange shaded areas represent the cloud of gluons.

$n \rightarrow p\, e^-\, \bar{\nu}_e$



A free neutron (udd) decays to a proton (uud), an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β (beta) decay.

$e^+ e^- \rightarrow B^0 \bar{B}^0$

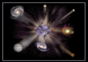


An electron and positron (antielectron) colliding at high energy can annihilate to produce $\bar{B}^0$ and B^0 mesons via a virtual Z boson or a virtual photon.

Unsolved Mysteries

Driven by new puzzles in our understanding of the physical world, particle physicists are following paths to new wonders and startling discoveries. Experiments may even find extra dimensions of space, microscopic black holes, and/or evidence of string theory.

Why is the Universe Accelerating?



The expansion of the universe appears to be accelerating. Is this due to Einstein's Cosmological Constant? If not, will experiments reveal a new force of nature or even extra (hidden) dimensions of space?

Why No Antimatter?

Matter and antimatter were created in the Big Bang. Why do we now see only matter except for the tiny amounts of antimatter that we make in the lab and observe in cosmic rays?

What is Dark Matter?

Invisible forms of matter make up much of the mass observed in galaxies and clusters of galaxies. Does this dark matter consist of new types of particles that interact very weakly with ordinary matter?

Are there Extra Dimensions?

An indication for extra dimensions may be the extreme weakness of gravity compared with the other three fundamental forces (gravity is so weak that a small magnet can pick up a paper clip overwhelming Earth's gravity).

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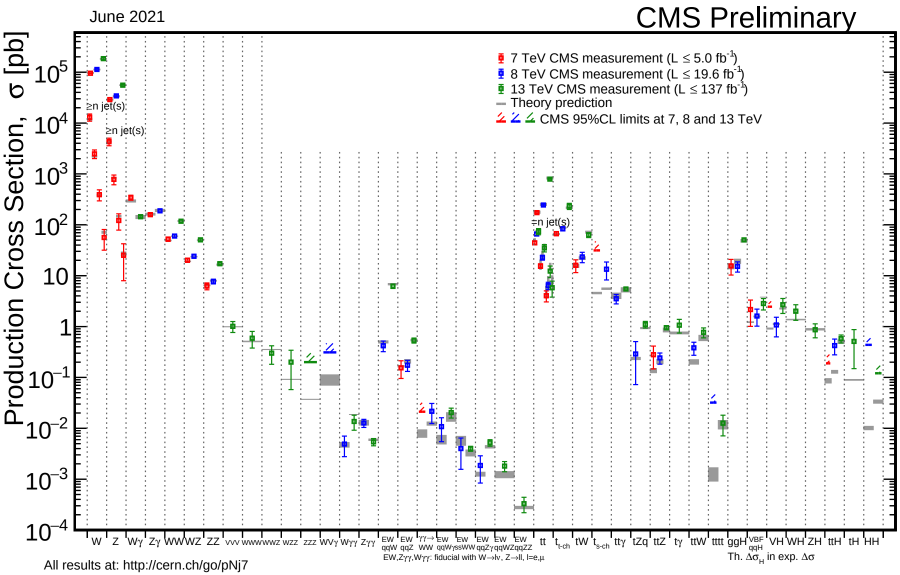
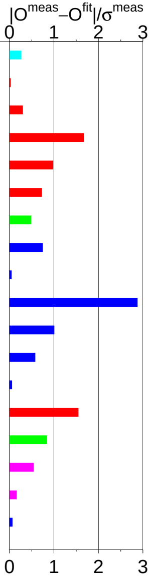


June 2021
CMS Preliminary
Production Cross Section, σ [pb]
7 TeV CMS measurement (L ≤ 5.0 fb-1)
8 TeV CMS measurement (L ≤ 19.6 fb-1)
13 TeV CMS measurement (L ≤ 137 fb-1)
Theory prediction
CMS 95%CL limits at 7, 8 and 13 TeV
≥n jet(s)
≥n jet(s)
=n jet(s)
W Z Wγ Zγ WW WZ ZZ VVV WWW WWZ WZZ ZZZ WVγ Wγγ Zγγ EW qqW EW qqZ γγ→WW EW qqWγ EW ssWW EW qqZγ EW qqWZ EW qqZZ tt t t-ch tW t s-ch ttγ tZq ttZ tγ ttW tttt ggH VBF qqH VH WH ZH ttH tH HH
EW,Zγγ,Wγγ: fiducial with W→lν, Z→ll, l=e,μ
Th. Δσ_H in exp. Δσ
All results at: http://cern.ch/go/pNj7

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2012

Quantum Field Theory (QFT)

QFT stands as our best tool for describing the fundamentals laws of nature.

- How does QFT improves our understanding of nature with respect to non-relativistic Quantum Mechanics and Classical Field Theory?
  - Dynamical degrees of freedom become operators that are functions of spacetime.
    - Quantum fields obey appropriate commutation relations.
  - Interactions of the fields are local – no "spooky action at a distance".
  - When combined with symmetry postulates (Lorentz, gauge), it becomes a powerful tool to describe interactions.
- Quantum Theory of **Free** Fields brings:
  - Existence of indistinguishable particles.
  - Existence of antiparticles.
  - Quantum statistics.
- Quantum Theory of **Interacting** Fields also brings:
  - The appearance of processes with creation and destruction of particles.
  - The association of interactions with exchange of particles.

From Particles to Fields (1)

Consider a state describing a “particle” of mass m and spin s (or helicity h):

$$|\vec{p}, s_z, \sigma\rangle \quad \begin{cases} \hat{\vec{P}}\,|\vec{p}, s_z, \sigma\rangle = \vec{p}\,|\vec{p}, s_z, \sigma\rangle \\ \hat{H}\,|\vec{p}, s_z, \sigma\rangle = E_{\vec{p}}\,|\vec{p}, s_z, \sigma\rangle\,, \text{ with } E_{\vec{p}} = \sqrt{p^2 + m^2} \\ \hat{S}_z\,|\vec{p}, s_z, \sigma\rangle = s_z\,|\vec{p}, s_z, \sigma\rangle\,, \text{ etc.} \end{cases}$$

σ stands for internal quantum numbers.

Lorentz invariance requires that the Hilbert space contain all state vectors for all momenta on the “mass shell”: $p^2 = p_\mu p^\mu = m^2$.

In addition, particle types are labeled by the total spin s.

Classification of the irreducible representations of the 4D Lorentz group acting on the Hilbert space states.

Exactly what you know about spin from quantum mechanics.

Particles can also carry other “internal charges” (e.g. electric charge).

From Particles to Fields (2)

A single particle in the universe is described by the state:

$$|\vec{p}, s_z, \sigma\rangle = a^\dagger_{\vec{p},s_z,\sigma}|0\rangle$$

Multi particle states and statistics:

$$\begin{aligned} &\text{Bose-Einstein (bosons)} \quad && \left[a_{\vec{p},s_z,\sigma}, a^\dagger_{\vec{p}',s'_z,\sigma'}\right] && = (2\pi)^3 2E_{\vec{p}}\, \delta^{(3)}\left(\vec{p}-\vec{p}'\right) \delta_{s_z,s'_z}\delta_{\sigma,\sigma'} \\ &\text{Fermi-Dirac (fermions)} \quad && \left\{a_{\vec{p},s_z,\sigma}, a^\dagger_{\vec{p}',s'_z,\sigma'}\right\} && = (2\pi)^3 2E_{\vec{p}}\, \delta^{(3)}\left(\vec{p}-\vec{p}'\right) \delta_{s_z,s'_z}\delta_{\sigma,\sigma'} \end{aligned}$$

whilst all other (anti-)commutators vanish.

Non-interacting particle states built by repeated application of creation operators.

Indistinguishable particles: states labeled by **occupation numbers**, i.e. how many quanta (particles) of a given momentum, z-spin, charge, etc.

From Particles to Fields (3)

Convenient to put all possible 1-particle momentum "states" together by Fourier transforming. To illustrate, we consider a spin-0 particle, and define

$$\Phi_+(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\vec{k}}} a_{\vec{k}} \; e^{-ik_\mu x^\mu} \Big|_{k_0 = E_{\vec{k}}}$$

If the particle carries a charge, the anti-particle is distinct, and we define

$$\Phi_-(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\vec{k}}} b^\dagger_{\vec{k}} \; e^{ik_\mu x^\mu} \Big|_{k_0 = E_{\vec{k}}}$$

One can then show from the commutation relations given earlier that the **field**

$$\Phi(x) \equiv \Phi_+(\vec{x}, t) + \Phi_-(\vec{x}, t)$$

obeys $\left[\Phi(x), \Phi(y)^\dagger\right] = 0$ for $(x - y)^2 < 0$ (spacelike separation). It is a **causal field**.

Note also that the field

$$\Phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\vec{k}}} \left\{ a_{\vec{k}} e^{-ik\cdot x} + b^\dagger_{\vec{k}} e^{ik\cdot x} \right\}\Big|_{k_0 = E_{\vec{k}}}$$

satisfies the Klein-Gordon equation, $\left(\partial_\mu \partial^\mu + m^2\right) \Phi(x) = 0$. The K-G equation encodes the relativistic energy-momentum equation, $E^2 = p^2 + m^2$, when one uses the prescription for the quantum-mechanical operators:

$$\vec{p} \to -i\vec{\nabla},\ E \to i\frac{\partial}{\partial t}$$

Notice that it can be derived from the Lagrangian density

$$\mathcal{L} = \partial_\mu \Phi(x)^\dagger \partial^\mu \Phi(x) - m^2 \Phi(x)^\dagger \Phi(x)$$

following the usual variational principle one learns in classical mechanics:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \Phi\right)} - \frac{\partial \mathcal{L}}{\partial \Phi} = 0$$

Other Free Lagrangians

- The Dirac Lagrangian:

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi$$

leading to the Dirac equation for spin-1/2 particles:

$$i\gamma^\mu\partial_\mu\psi - m\psi = 0$$

- The free EM Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \text{ where } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

leading to “Maxwell's equations” for the free EM field:

$$\partial_\mu F^{\mu\nu} = 0$$

Interactions (1)

In principle, we could choose any form for our interactions. The form of the potential in Schrödinger's equation is arbitrary... but let's take a closer look at electromagnetism.

The electric and magnetic fields can be described in terms of $A^\mu = (\phi, \vec{A})$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}; \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

that are invariant under the **gauge transformation**:

$$\phi \to \phi' = \phi - \frac{\partial \chi}{\partial t}, \quad \vec{A} \to \vec{A'} = \vec{A} + \vec{\nabla}\chi$$

At this level, this is a useful property that helps us solve EM problems in terms of the potentials. Choosing the right gauge can immensely simplify the equations for $\phi, \vec{A}$.

Gauge Invariance in Quantum Mechanics

Let us consider the classical Hamiltonian that gives rise to the Lorentz force:

$$H = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + q\phi$$

With the usual operator prescription ($\vec{p} \to -i\vec{\nabla}$, $E \to i\partial_t$) we get the Schrödinger equation for a particle in an electromagnetic field:

$$\left[\frac{1}{2m}(-i\vec{\nabla} - q\vec{A})^2 + q\phi\right]\psi(x,t) = i\frac{\partial\psi(x,t)}{\partial t}$$

which can be written as:

$$\frac{1}{2m}(-i\vec{D})^2\psi = iD_0\psi, \text{ with } \begin{cases} \vec{D} = \vec{\nabla} - iq\vec{A} \\ D_0 = \frac{\partial}{\partial t} + iq\phi \end{cases}$$

On the other hand, if we take the **free** Schrödinger equation and make the substitution

$$\vec{\nabla} \rightarrow \vec{D} = \vec{\nabla} - iq\vec{A}, \quad \frac{\partial}{\partial t} \rightarrow D_0 = \frac{\partial}{\partial t} + iq\phi$$

we arrive at the same equation.

Now, if we make the gauge transformation $(\phi, \vec{A}) \xrightarrow{G} \left(\phi', \overrightarrow{A'}\right)$ does the solution of

$$\frac{1}{2m}\left(-i\vec{D}'\right)^2 \psi' = iD_0'\psi'$$

describe the same physics?

No! We need to make a phase transformation on the matter field:

$$\psi' = \underbrace{\exp(iq\chi)}_{U(1)\ \text{transformation}} \psi$$

with the same $\chi = \chi(x, t)$. The derivatives transform as:

$$
\begin{aligned}
\vec{D}'\psi' &= \left[\vec{\nabla} - iq(\vec{A} + \vec{\nabla}\chi)\right]\exp(iq\chi)\psi \\
&= \exp(iq\chi)(\vec{\nabla}\psi) + iq(\vec{\nabla}\chi)\exp(iq\chi)\psi - iq\vec{A}\exp(iq\chi)\psi - iq(\vec{\nabla}\chi)\exp(iq\chi)\psi \\
&= \exp(iq\chi)\vec{D}\psi, \\
D_0'\psi' &= \exp(iq\chi)D_0\psi
\end{aligned}
$$

The Schrödinger equation now maintains its form, since:

$$
\begin{aligned}
\frac{1}{2m}(-i\vec{D}')^2\psi' &= \frac{1}{2m}(-i\vec{D}')(-i\vec{D}'\psi') \\
&= \frac{1}{2m}(-i\vec{D}')\left[-i\exp(iq\chi)\vec{D}\psi\right] \\
&= \exp(iq\chi)\frac{1}{2m}(-i\vec{D})^2\psi \\
&= \exp(iq\chi)\,(iD_0)\,\psi = iD_0'\psi'
\end{aligned}
$$

whilst **both fields describe the same physics** since $|\psi|^2 = |\psi'|^2$.

In order to make all variables invariant we should substitute

$$\vec{\nabla} \to \vec{D}, \qquad \frac{\partial}{\partial t} \to D_0$$

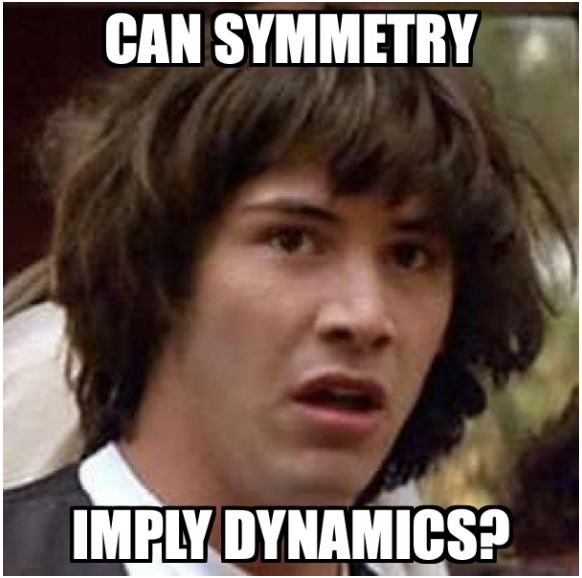
and the current $\vec{J} \sim \psi^*(\vec{\nabla}\psi) - (\vec{\nabla}\psi)^*\psi$ also becomes gauge invariant:

$$\psi^{*\prime}\left(\vec{D}'\psi'\right) = \psi^* \exp(-iq\chi)\exp(iq\chi)(\vec{D}\psi) = \psi^*(\vec{D}\psi)$$

Could we reverse the argument?

> When we demand that a theory is invariant under a space-time dependent phase transformation, can this procedure impose the specific form of the interaction with the gauge field?

In other words. . .


CAN SYMMETRY
IMPLY DYNAMICS?

Quantum Electrodynamics (QED) – Our Best Theory

Start from the free electron Lagrangian

$$L_e = \overline{\psi}\left(i\gamma_\mu \partial^\mu - m\right)\psi$$

Impose **invariance** under local phase transformation:

$$\psi \rightarrow \psi' = \exp[i\alpha(x)]\psi$$

Introduce the photon field and the coupling via **covariant derivative**

$$D_\mu = \partial_\mu + ieA_\mu \qquad A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

This determines the interaction term with the electron:

$$L_{\text{int}} = -e\overline{\psi}\gamma_\mu \psi A^\mu$$

Introduce the free photon Lagrangian:

$$L_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and the QED Lagrangian comes out as:

$$L_{\text{QED}} = \overline{\psi}\left(i\gamma_\mu \partial^\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\overline{\psi}\gamma_\mu \psi A^\mu$$

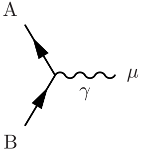
Notice the absence of photon mass terms $\frac{1}{2}m^2 A_\mu A^\mu$. They are forbidden for they break the gauge symmetry.

Interactions (2)

Focus on the interaction we have proposed:

$$-e\underbrace{\overline{\psi}\gamma^\mu\psi}_{J^\mu}A_\mu = -e\overline{\psi}^A(\gamma^\mu)_A{}^B\psi_B A_\mu$$

This is an interaction between one photon, and two electrons. It is conveniently represented by



$$= \quad -ie(\gamma^\mu)_A{}^B$$

This is of course just a simplification! In QFT, you learn how to calculate two-point correlation functions in perturbation theory, use Wick's theorem and write **Feynman diagrams**!

Interpretation: $A_\mu \sim a + a^\dagger \qquad \overline{\psi} \sim b^\dagger + c \qquad \psi \sim b + c^\dagger$

It leads to transitions like $\underbrace{\langle\gamma|a^\dagger bc|e^+e^-\rangle}_{e^+e^-\to\gamma}$ or $\underbrace{\langle\gamma e^-|a^\dagger b^\dagger b|e^-\rangle}_{e^-\to e^-\gamma}$

Strictly speaking, need a fourth particle to absorb momentum, but can occur as a "virtual" process.

Testing QED – Anomalous Magnetic Dipole Moment

Back to Dirac's equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[c\vec{\alpha}.\left(\vec{p} - \frac{e}{c}\vec{A}\right) + \beta mc^2 + e\phi\right]\psi$$

Get the Pauli equation for the "large component" of the spinor:

$$i\hbar\frac{\partial\xi}{\partial t} = \left[\frac{\vec{p}^2}{2m} - \frac{e}{2mc}(\vec{L} + 2\vec{S})\cdot\vec{B}\right]\xi$$

where the red coeficient – interaction between the spin and the magnetic field – is called the gyromagnetic factor g_e. The anomalous magnetic dipole moment a_e is defined by:

$$a_e = \frac{g_e - 2}{2}$$

Pauli's theory is the first-order prediction ("tree level"), $a_e = 0$. Dirac's theory predicts higher-order contributions ("loops") and a non-zero a_e.

The anomalous magnetic dipole moment receives, in principle, contributions from all interactions:

$$a_e = a_{\mathrm{QED}} + a_{\mathrm{EW}} + a_{\mathrm{HAD}} + a_{\mathrm{NEW}}$$

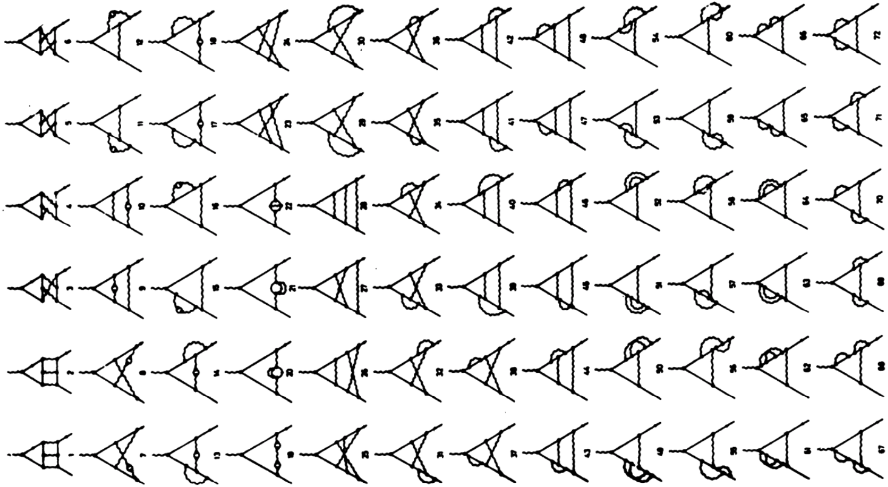
QED's contribution can be written as a series in (α/n):

$$a_{\mathrm{QED}} = \sum_{n\geq 1} A_n(\ell)\left(\frac{\alpha}{\pi}\right)^n + \sum_{n\geq 2} B_n\left(\ell,\ell'\right)\left(\frac{\alpha}{\pi}\right)^n$$

The dimensionless coefficients A_n are universal – they don't depend on the lepton flavour. Some calculations:

$$\begin{aligned}
A_1 &= +0.5 & & \\
A_2 &= -0.328478965 & & 7 \text{ diagrams, } 1950(\mathrm{W}), 1958 \\
A_3 &= +1.181241456 & & 72 \text{ diagrams, } 1996 \\
A_4 &= -1.91298(84) & & 891 \text{ diagrams, } 2003 \\
A_5 &= +7.795(336) & & 12672 \text{ diagrams, } 2014
\end{aligned}$$

The 72 Feynman diagrams that contribute to A_3:



Anomalous Magnetic Dipole Moment – Experiment

To measure a_e, one uses a Penning trap – a magnetic trap at low temperatures. The spin flip frequency for a given magnetic field is related to g_e

0.00119(5)	4.2%	1947
0.001165(11)	1%	1956
0.001116(40)	3.6%	1958
0.0011609(24)	2100 ppm	1961
0.001159622(27)	23 ppm	1963
0.001159660(300)	258 ppm	1968
0.0011596577(35)	3 ppm	1971
0.00115965241(20)	172 ppb	1977
0.0011596521884(43)	4 ppb	1987

$$a_e^{\text{theory}} = 0.001\ 159\ 652\ 181\ 643\ (763)$$
$$a_e^{\text{exper.}} = 0.001\ 159\ 652\ 180\ 73\ (28)$$

Agreement of nine significant digits!

But What About The Muon $g - 2$???

All of the above is also true for the muon.

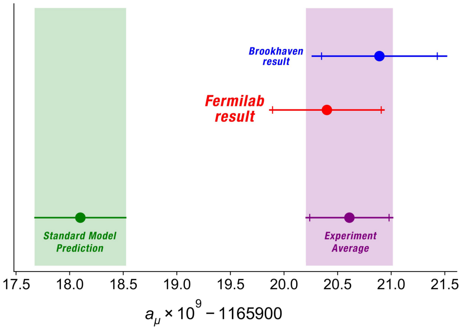
- Assuming lepton universality.

207 times as massive as the electron.

- Particularly sensitive to new types of virtual particles.

4.2σ difference between theory and experiment. (Fermilab's $g - 2$ and Brookhaven combined)

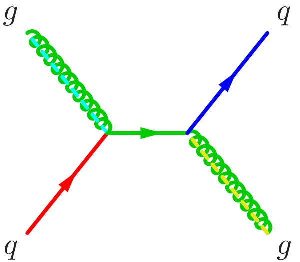
$$a_\mu^{\text{theory}} = 0.001\,165\,918\,10\,(43)$$
$$a_\mu^{\text{exper.}} = 0.001\,165\,920\,61\,(41)$$



- 1+2 gauge interactions:
  - $SU(3)_C$ strong (a.k.a. QCD).
  - $SU(2)_L \times U(1)_Y$ electroweak (EW).
  - Electroweak symmetry breaking (EWSB): weak interactions and EM are observed as separated phenomena at low energies.
- Two kinds of matter particles:
  - Quarks subject to all three interactions.
  - Leptons subject to EW interaction only.
- Gauge mediators:
  - Photon (γ) for the electromagnetism.
  - W^+, W^-, Z^0 for the weak interaction.
  - Gluon (g) for the strong interactions.
- Scalar field (ϕ) / Higgs boson (H).

Basics of QCD

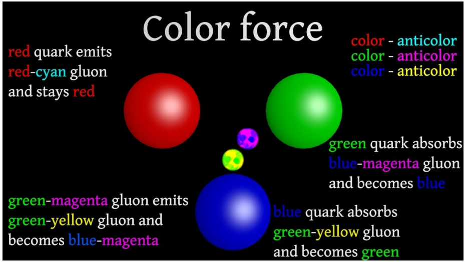
- Symmetry group is $SU(3)_C$
- Quarks come in three colors: **R**, **G**, **B**
  - They **transform** under the fundamental representation of $SU(3)$ – a triplet.
- The quantum of the gauge field is the gluon, and it comes in eight bicolored varieties (color $+$ anticolor).
  - They **transform** under the adjoint representation of $SU(3)$ – the eight generators λ_i.
  - Since the gluons carry color themselves, they can self-interact – there are qqg, ggg and gggg vertices in the theory. Compare with the single eeγ vertex in QED.
- The theory is **renormalizable**!
  - When making higher-order calculations in QFT, we encounter divergences.
  - **Renormalization** is a collection of techniques to address those divergences.
  - **Observables** remain finite (renormalized); "bare" parameters in $\mathcal{L}$ are formally infinite.
    - In QFT we also learn how to do it with renormalized parameters from the start.
  - A non-renormalizable theory is not amenable to standard perturbative calculations...
- A price to pay: coupling constant α_S depends on interaction energy scale Q.



Almost like calculating $e\gamma \to e\gamma$ but considering a colour factor.

$$\underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\frac{1}{2}(\lambda_1 - i\lambda_2)} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

The $\overline{\text{R}}$G gluon transforms the R quark into a G quark.



From IEET2 Youtube Channel

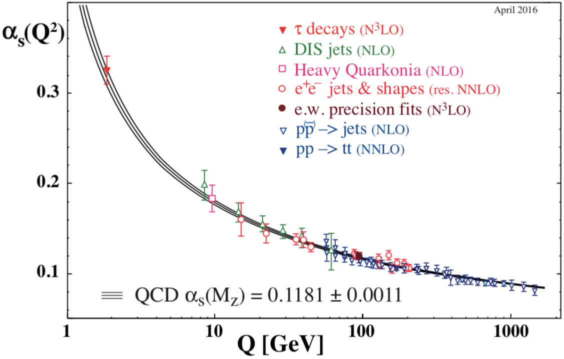
QCD Running Coupling – Asymptotic Freedom and Confinement

- The presence of gluon self-interactions (ggg), in addition to the qqg vertex, leads to an expression for $\alpha_S(Q^2)$:

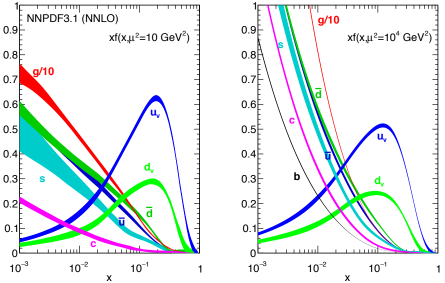
$$\alpha_s\left(Q^2\right) = \frac{\alpha_s\left(\mu^2\right)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}\left(33 - 2n_f\right)\log\left(Q^2/\mu^2\right)}$$

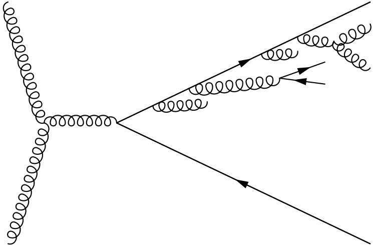
  where n_f is the number of flavours, and μ^2 is the renormalization scale.

- **Asymptotic freedom:** for high Q^2 (short distances), α_s becomes very small $\Rightarrow$ quarks become quasi-free.
- **Confinement:** for low Q^2, α_s becomes very large $\Rightarrow$ no isolated quarks.
  - Hadrons – colorless bound states. Either mesons ($q\bar{q}$) or barions (qqq).



PDFs, Showering, Hadronization, Jets





- When you calculate a process via Feynman diagrams, you assume that the initial and final states are free particles. . . but there are no free q's or g's!
- Quarks and gluons – partons – are bound inside hadrons, but in that state they are quasi-free! The **parton distribution functions** $u^p(x)$ give the probability of having a parton of type u inside the proton.
- Final state q's and g's radiate / branch, and their energy gets diluted in a **parton shower**. The branchings are primarily soft and collinear – after a given point the process has to be treated non-perturbatively (high α_S).
- Eventually, the whole system changes phase into a set of hadrons. Hadrons that come from a parton keep its original direction, forming a **hadronic jet**.

Basics of the Electroweak Model

- Symmetry group is $SU(2)_L \times U(1)_Y$.
- Quarks and leptons come in six flavours: u, d, c, s, t, b; e, ν_e, μ, ν_μ, τ, ν_τ.
- Particles have definite **chirality**: **transform** in a right-handed (R) or left-handed (L) representation of the Poincaré group. For massless particles chirality $\Leftrightarrow$ helicity $\sim$ sign$(\vec{s} \cdot \vec{p})$.
- Left-handed particles ψ_L form a weak isospin doublet, $(\uparrow, \downarrow)$. Right-handed particles ψ_R are weak isospin singlets. All particles have also a hypercharge Y.
- The quantum of the $SU(2)_L$ gauge field are the weak bosons W_1, W_2, W_3; for the $U(1)_Y$ field it is the B boson.

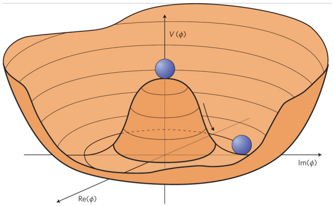
...and this has nothing to do with the real particles we talked about previously! Notice that:

- The Lagrangian can't have fermion mass terms: $\overline{\psi}\psi = \overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R$ has mixed symmetry.
- The W_i, B bosons are massless, whilst the weak bosons are massive.

Electroweak Symmetry Spontaneous Breaking

- Add to the Lagrangian a complex scalar field ϕ:

$$\mathcal{L}_{\text{scalar}} = |D_\mu \phi|^2 - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2, \text{ with } \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$



  - ϕ is an $SU(2)_L$ doublet , with hypercharge suitably chosen.
- Choose μ, λ such that the **vacuum expectation value $\boldsymbol{v}$ of ϕ is not zero**.
- The ground state of ϕ is now asymmetric, but the system as whole still is. The $SU(2)_L$ symmetry is broken (hidden).
- Rewrite ϕ as:

$$\phi(x) = \exp\left[i\frac{\sigma_i}{2}\theta^i(x)\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \boldsymbol{v} + H(x) \end{pmatrix},$$

and rewrite $\mathcal{L}$ substituting ϕ for $\phi(H, \theta_i; v)$ and mass terms appear for the W and Z bosons.

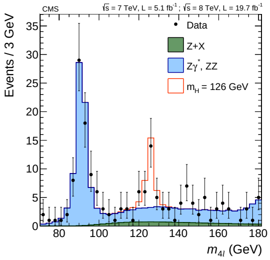
- Technicality: do some field redefinitions to make $W^{\pm}, Z, \gamma$ appear from the W_i, B fields.
- Technicality: use the gauge symmetry to end up only with $\phi(H; v)$; the θ_i fields disappear.

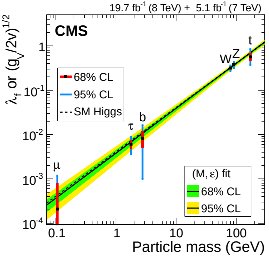
I.e., all terms of the form $\phi^2 VV$, with $V = W, Z$, give rise to terms $v^2 VV$; v is a constant, so these are mass terms for the bosons.

- But wait, weren't mass terms forbidden by the symmetry?
- That's the thing, the symmetry is hidden ("spontaneously broken") by the particular vacuum configuration we are in!
- The value of $v \sim 246$ GeV can be inferred from low-energy physics.

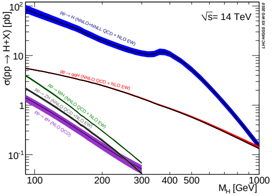
Yukawa couplings of the form $\phi\overline{\psi}\psi$ give mass to the fermions as well.

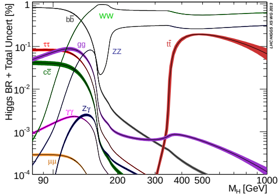
The Higgs Boson





- One last field $H(x)$ remains in the theory after EWSB. Its quantum is the Higgs boson.
- Its mass is **not fixed** from low-energy physics.
  - Fine structure α, Fermi's G_F, Weinberg angle θ_W fix all other terms in the Lagrangian.
- Higgs properties are exquisitely dependent on its mass.
- Discovery on July 4th, 2012 by the ATLAS and CMS collab.
- All properties as expected by the SM, $m_{\mathrm{H}} = 125.2\,\mathrm{GeV}$.

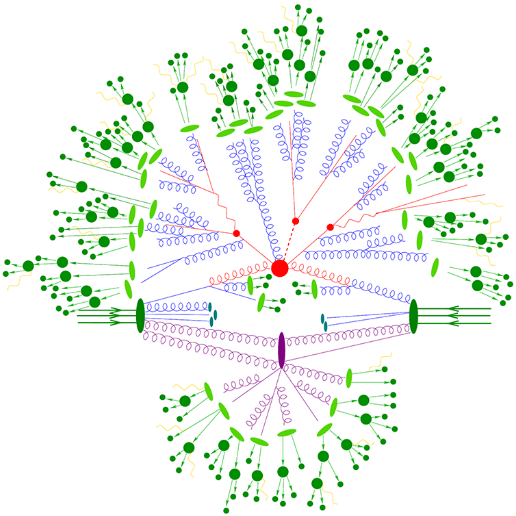




High-Energy Hadron Collisions

Full recipe for calculations

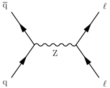
- ☐ Calculate hard matrix elements from perturbative QFT
- ☐ Embed initial state partons in protons via structure functions
- ☐ Add corrections for higher-order + non-perturbative processes to the process.
  - Initial and final-state radiation
  - Underlying event (i.e. "what happens to the rest of the hadron?")
  - Hadronisation and decays of unstable particles



Feynman Rules

FeynRules is a Mathematica®-based package which addresses the implementation of particle physics models, which are given in the form of a list of fields, parameters and a Lagrangian, into high-energy physics tools.

Matrix Element Calculations



CalcHEP is a package for the automatic evaluation of production cross sections and decay widths in elementary particle physics at the lowest order of perturbation theory.

MadGraph5_aMC@NLO is a framework that aims at providing all the elements necessary for HEP phenomenology: cross-section computations, hard events generation and matching with shower codes.

Parton Shower and Hadronisation

Herwig is a general-purpose Monte Carlo event generator for the simulation of hard lepton-lepton, lepton-hadron and hadron-hadron collisions.

Pythia is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state.

Data Formats

- UFO: The Universal FeynRules Output (link)
- LHE: A standard format for Les Houches Event Files (link)
- HepMC: an object oriented, C++ event record for High Energy Physics Monte Carlo generators and simulation (link)

Enough theory for now. . .

. . . let's start detecting those particles!